PH3010-Advanced Skills 2nd Report

Phenomenological and Theoretical structure of Higgs boson decays

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Abstract

For centuries one question was always crucial for the human being: what we are we made of? From ancient Greeks to recent Era we have achieved a great understanding of this Universe, our achievement in understanding what we are made of lies in Physics, particularly in the Standard Model. The Higgs mechanism and its boson are crucial for the standard model. In this report lies the basic fundamental understanding of this phenomenon. By building and describing the theoretical decay channels of the Higgs boson it is possible to plot the branching ratio. Having an overlook of how it behaves concluding with a theoretical derivation of the bosonic decay.

1 Introduction

One of the biggest mysteries of our existence is about what we are made of, from the ancient greeks until now humanity have has been searching the answer. Today the goal of particle physics is the experimental confirmation of the theory that holds up the Standard Model (SM). The SM has been proved with many experiments through the last century. One of the keys of the research today is the description of the elementary particle masses. This can be achieved via the Higgs mechanism, which predicts the existence of the Higgs boson. The experiments illustrate a particle has been found that looks like the higgs boson, which has no spin and even parity, two fundamental criteria of the Higgs boson consistent with the SM. If we couple this evidence with the measured interactions of this new particle with others, one can conclude that this new particle strongly indicates that it is a Higgs boson. Following this assumptions, the various decay channels of the Higgs boson can be studied to learn more about its interaction in the SM. Two main characteristic are predictable in the Higgs decay: the branching ratio and the decay width. The decay width in particle physics is the probability per unit time that the particle will decay, however by looking at each individual decay of the Higgs boson it is possible to analyze the partial decay width, Instead the branching ratio is the probability of a particle decaying through a certain decay channel i, as shown below:

$$BR_i = \frac{\Gamma_i}{\Gamma_{total}}$$

where Γ_{total} is the sum of all partial decays of a particular particle, in this case the Higgs boson. In the next section some important knowledge of what is and how the Higgs mechanism behaves is stated, describing how the spontaneous broken symmetry works. In section 2 the partial decay widths are plotted against the mass of the Higgs boson (M_H), repeated for the different possible Higgs boson decays, Instead in section 3 all the partial decays of section 2 are reunited, and finally the branching ratio is plotted against the the M_H to have a complete overview of its fenomena. Finally in section 4 the partial decay width of higgs boson decaying in gauge bosons has been theoretically derived, where the same formula for two decays in section 2 have been used.

1.1 The Higgs Mechanism

A very essential part of the SM of particle physics is Quantum Field Theory (QFT). through this theory we can explain the property of "mass" for gauge bosons, this is possible thanks to Higgs mechanism.

In QFT every particle have has its own field, leading many problem through this scenario. Symmetry in physics it is very useful and important concept, used to simplify and understand this universe. It is vastly applied to QFT leading to many possible theories that can support the SM or create new models. A possible symmetry is found in gauge theories and global symmetry, but violated in different cases. As in magnet field theory this global field might have a directional character leading to a violation of the symmetry of the Lagrangian. In this scenario one may say that the field theory has a *spontaneously broken* symmetry. Spontaneous symmetry breaking can be defined as a spontaneous process, in a physical system where it is in a symmetric state ends up in a asymmetric state (see Ref.[2]). Without this spontaneous broken symmetry there would be no Higgs mechanism, meaning that all bosons would be considered massless. This case has been proved wrong as the measurement show that W^+, W^- and Z bosons actually have heavy masses.

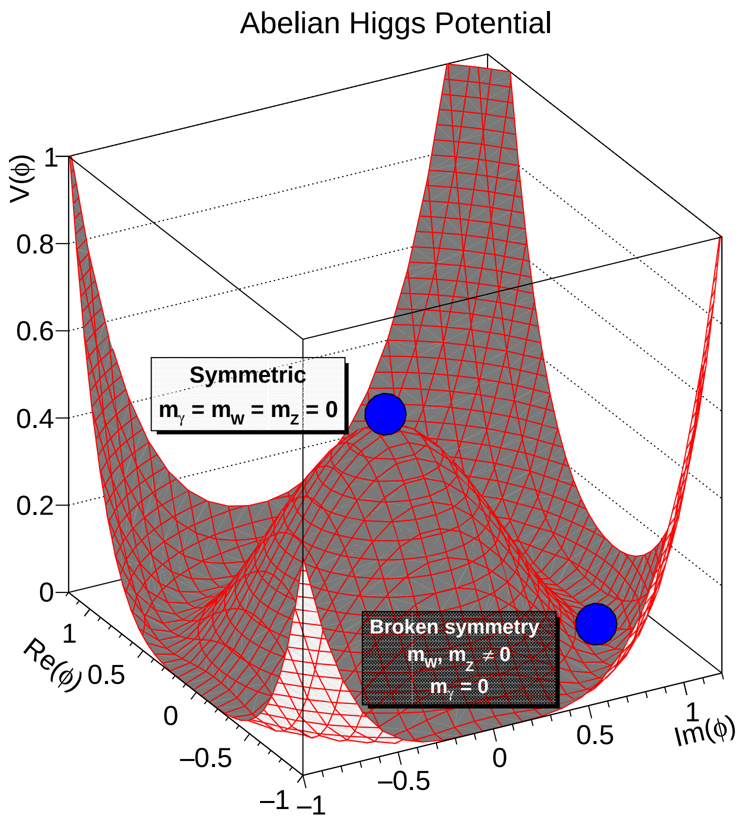


Figure 1: The Abelian V potential from Eq.2, setting λ to a constant and $\phi = 1$. Showing the spontaneus broken symmetry at the bottom and in the middle the symmetry is preserved.

Therefore, gauge theories with spontaneous symmetry breaking are the key to understanding how the higgs mechanism works. An useful example that can lead to understand these concepts is through the Abelian example. Consider a complex scalar field coupled to itself and to an electromagnetic field:

$$S = \int -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi, \Phi) \tag{2}$$

where the first term is the $U(1)$ gauge invariant kinetic term, $D_\mu = \partial_\mu + ieA_\mu$ is the extension of the model introducing a complex scalar field coupled to itself and a electromagnetic field. Finally the potential is $V(\phi, \Phi) = \lambda(|\phi|^2 - \Phi^2)^2$. The potential is key to understand where the spontaneous symmetry breaking happens, the minimum of the potential is obtained when the complex field ϕ is equal to Φ. If the potential V is nonzero, the gauge field acquires a mass. In low energy levels as shown in Fig. 1, the potential V assumes this form often called "Mexican hat", the form overall remains symmetric, but this form enforces the "ball" (in blue) to spontaneously rest at some spot in the bottom, resulting in spontaneously breaking the symmetry, as if the z-axis is divided in half, then the ball would be in one of the quadrant. This mechanism where the spontaneous symmetry breaking generates the mass for a gauge boson has been generalized in a non-Abelian case by Higgs, Kibble, Guralnik, Heagen, Brout and Englert, where now is known as the Higgs mechanism (see Ref.[1]).

2 Higgs decays

In this section the different types of higgs decays showed in the introduction have been shown, where in each section Feynman diagrams and plots of the different decays have been analysed, concluding with the result. In Section 2.1 2.2 and 2.3 the possible types of higgs decays are explained, showing feynman diagrams for each decay and plots of the partial decay width against the mass of the higgs boson.

2.1 Fermionic tree-level decay modes

In the fermionic tree-level decay modes there are four decay modes which have high probability to occur, as the light fermion such as $[u, d, s, e^-, \mu]$ due to their light mass requires high energy to produce their decays, therefore it is not most likely to appen. Below are written the most probable four decays:

$$H \to c\bar{c} \quad , \quad H \to b\bar{b} \quad , \quad H \to t\bar{t} \quad , \quad H \to \tau^+\tau^- \tag{3}$$

Therefore the Feynman diagram can be drawn for each decay and its shown in below:

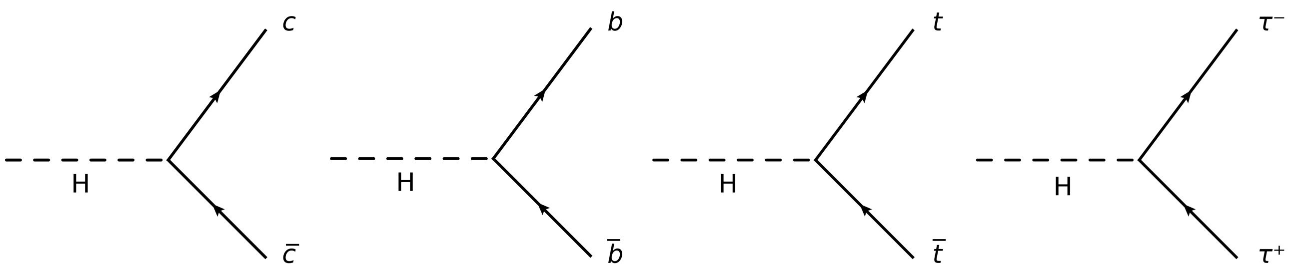


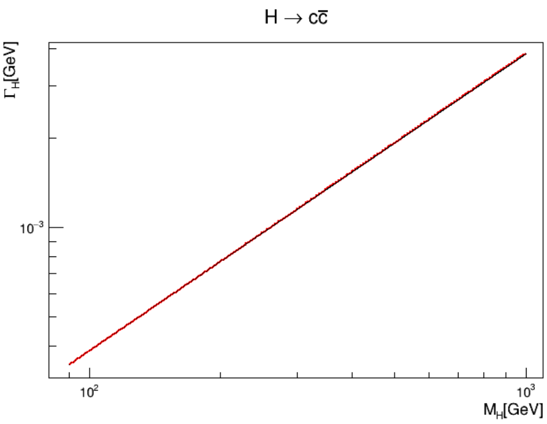
Figure 2: Correspective Feynman diagrams for each decay sh$\Gamma(H \to \gamma\gamma)$ vs $M_H$owed in Eq.3 of the most probabile fermions

The analytic formula that describes the tree levels decay rate for Higgs boson decaying in the corrispective fermions can be found in Ref.[3], and it is showed below:

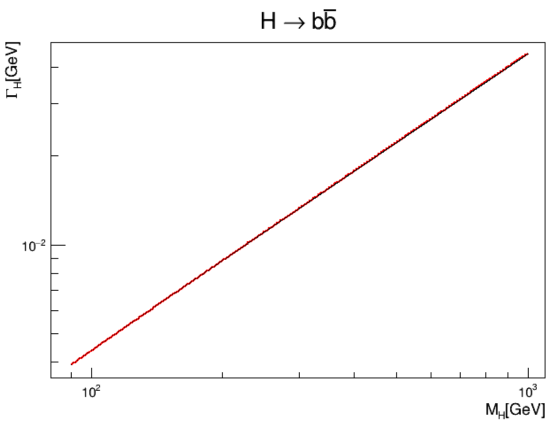
$$\Gamma(H \to f\bar{f}) = \frac{G_F M_H}{4\sqrt{2}\pi} N_c^f m_f^2 \beta_f^3 \tag{4}$$

where $\beta_f = \sqrt{1-\tau_f}$, $\tau_f = 4m_f^2/M_H^2$, and $(N_c)^{l,Q} = 1, 3$ where l and Q stand for leptons and quarks. G_F is the Fermi coupling constant, M_H is mass of the Higgs boson and m_f is the mass of the fermion.

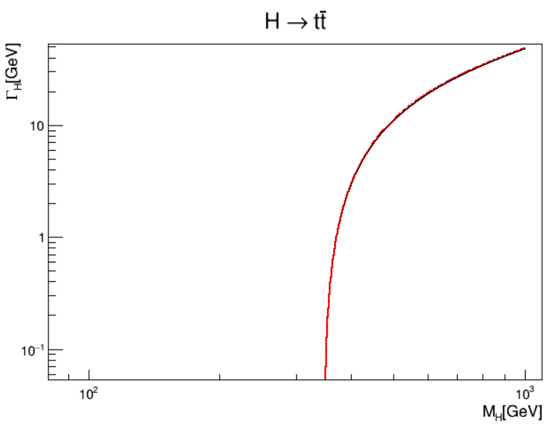
Using the function in Eq.4 acting on M_H for each decay shown in Eq.3 one can plot the respective 4 plot for each decay as shown below:



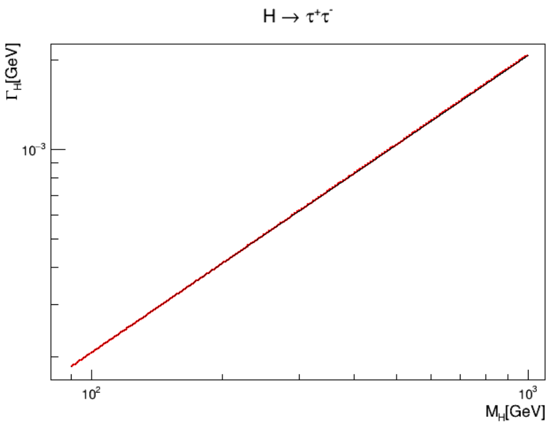
(a) $\Gamma(H \to c\bar{c})$ vs M_H



(b) $\Gamma(H \to b\bar{b})$ vs M_H



(c) $\Gamma(H \to t\bar{t})$ vs M_H



(d) $\Gamma(H \to \tau^+\tau^-)$ vs M_H

Figure 3: Decay width against M_H in the case of fermionic tree-level decay modes

It is important to mention that Eq.4 do not include Quantum Chromodynamics (QCD) correction, this means that it is affecting the plots but not drastically.

One may observe that $H \to t\bar{t}$ have a different pattern as the other 3 plots shown in Fig. 3. This is due because the top quark have a heavy mass which, compared to other fermions, they are much lighter. It is possible to denote that the top quark is so heavy that the Higgs mass must be greater than the double of the top quark mass for decaying, as it is shown in plot (c) Fig. 3.

2.2 Bosonic tree-level decay modes

In the Bosonic decay only two decays are possible, as shown below:

$$H \to W^+W^- \quad , \quad H \to ZZ \tag{5}$$

These decays can be represented with the feynman diagrams, as shown below :

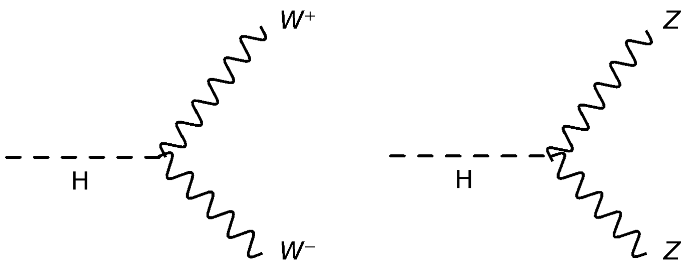


Figure 4: Correspective Feynman diagrams for each decay showed in Eq.5

Since the bosons are heavy particles with a mass around 80 MeV, therefore when the Higgs boson is below the mass of each boson will decay in 3 or 4 particles. This is because the Higgs boson does not have enough mass and therefore will decay in the respective virtual gauge boson (W, Z), which, in turn, will decay in other particles. Therefore, before the threshold, when the Higgs mass cannot give enough mass for the gouge bosons, this type of decay is called off-shell. However, when the M_H have enough mass for the bosons, the decay is called on-shell. This leads to different functions for the on-shell and off-shell decays that are described in the next section and can be found in Ref.[3]

2.2.1 Functions for the Off-shell & On-shell partial decay width

The partial decay width for the Off-shell region for $H \to VV* \to Vf_i\bar{f}_j$ is given by:

$$\Gamma(H \to WW^*) = \frac{3g^4 M_H}{512\pi^3} F\left(\frac{M_W}{M_H}\right), \tag{6}$$

$$\Gamma(H \to ZZ^*) = \frac{3g^4 M_H}{2048(1-s_W^2)^2\pi^3}\left(7 - \frac{40}{3}s_W^2 + \frac{160}{9}s_W^4\right) F\left(\frac{M_Z}{M_H}\right), \tag{7}$$

where $s_W = \sin\theta_W$ and θ is the Weinberg angle and the function $F(x)$ is given by

$$F(x) = -(1-x^2)\left(\frac{47}{2}x^2 - \frac{13}{2} + \frac{1}{x^2}\right) - 3(1-6x^2+4x^4)\ln(x) + 3\frac{1-8x^2+20x^4}{\sqrt{4x^2-1}}\arccos\left(\frac{3x^2-1}{2x^3}\right). \tag{8}$$

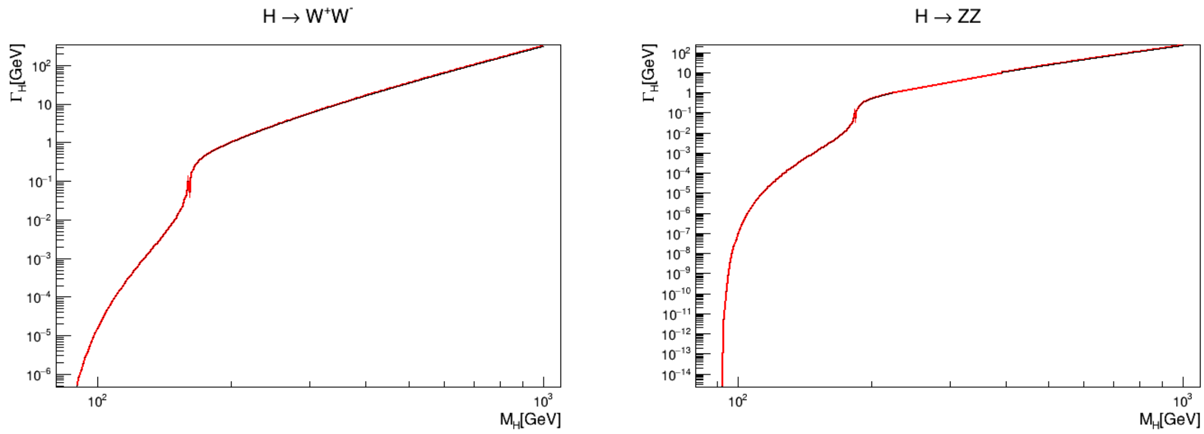
Instead the tree-level decay for the On-shell region for $H \to VV$ $(V = W^{\pm}, Z)$ can be written as:

$$\Gamma(H \to VV) = \frac{G_F M_H^3}{16\sqrt{2}\pi}\delta_V \left(1 - \tau_V + \frac{3}{4}\tau_V^2\right)\beta_V, \tag{9}$$

where $\beta_V = \sqrt{1-\tau_V}$, $\tau_V = 4M_V^2/M_H^2$ and $\delta_{W,Z} = 2, 1$ for the each case of the boson.

2.2.2 Plots of the decays

Taking these equations is possible to plot the Off-shell region and the On-shell region in the same axis for each decay written in Eq.5, in below the result:



(a) $\Gamma(H \to WW^*) + \Gamma(H \to VV)$

(b) $\Gamma(H \to ZZ^*) + \Gamma(H \to VV)$

Figure 5: Decay width against M_H for bosonic tree-level decay modes where $V = W^{\pm}, Z$

It is possible to denote the threshold region discussed before, in both plots in Fig.5 denoted by the kink. In this region the mass of the Higgs boson happens to be double of the each gauge bosons. For the decay of Higgs boson in W boson the threshold happens around the double of his mass $M_H \approx 2M_W \approx 160$ GeV, the same for the case of the other boson $M_H \approx 2M_Z \approx 182$ GeV. This is reasonable as discussed before, because before the threshold the Higgs Boson does not have sufficient mass to decay in the corrispective gauge bosons leading to a virtual boson.

2.3 Loop-induced decay modes

Another possible way in which the Higgs boson can decay is through loop-induced decays. Below the possible decays are shown:

$$H \to gg \quad , \quad H \to \gamma\gamma \quad , \quad H \to \gamma Z \tag{10}$$

the Higgs boson does not decay directly into photons or gluons. This is due to their lack of mass. Therefore, only medium virtual particles it is possible to achieve these decays. A useful Feynman diagram is shown below:

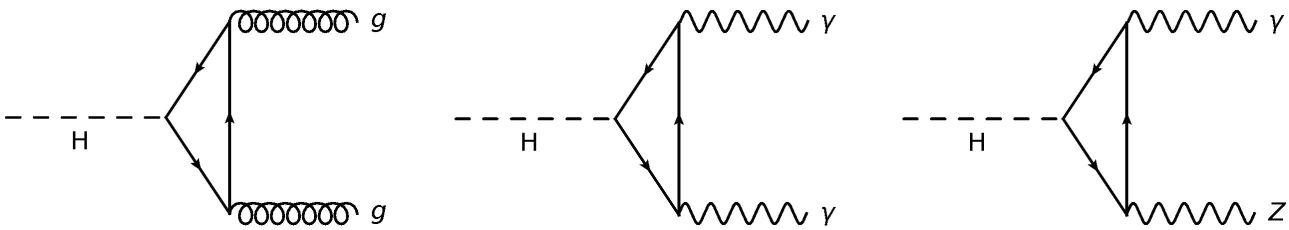


Figure 6: Correspective Feynman diagrams for each decay showed in Eq. 10

The loops in the Feynman diagrams are shown as triangle loop of "fermions", but the decay can actually be made through other particles. The most probable particles that can contribute to this decay are the heavy top quark and the W boson. In the case of Higgs boson decaying in gluons, only quarks can be the mediating particles in the loop. Instead, for the other two decays (the last two showed in Eq.10) can be mediated also by the W boson.

2.3.1 Function for the loop-decay modes

The main three principal functions used for the corrispective three decays are shown below:

- for $H \to gg$:

$$\Gamma(H \to gg) = \frac{G_F \alpha_s^2 M_H^3}{36\sqrt{2}\pi^3} \left| \frac{3}{4} \sum_q A_q^H(\tau_q) \right|^2 , \tag{11}$$

  where $\tau_q = 4m_q^2/M_H^2$.

- for $H \to \gamma\gamma$:

$$\Gamma(H \to \gamma\gamma) = \frac{G_F \alpha_s^2 M_H^3}{128\sqrt{2}\pi^3} \left| \sum_f N_c^f Q_f^2 A_f^H(\tau_f) + A_W^H(\tau_W) \right|^2 , \tag{12}$$

  where $N_c^f = 1, 3$ (for $f = l, q$ respectively), Q_f is the charge of the fermion secies, $\tau_f = 4m_f^2/M_H^2$.

- for $H \to \gamma Z$:

$$\Gamma(H \to \gamma Z) = \frac{G_F^2 M_W^2 \alpha M_H^3}{64\pi^4} \left(1 - \frac{M_Z^2}{M_H^2}\right)^2 \left| \sum_f A_f^H(\tau_{f}, _f) + A_W^H(\tau_{W}, _W) \right|^2 , \tag{13}$$

  where $\tau_i = 4M_i^2/M_H^2$ and $\lambda_i = 4M_i^2/M_Z^2$ (for $i = f, W$ respectively).

The principal 3 function for the 3 decays are stated in the last 3 equation. However, these equation use others equation to be complete. These equations are not fundamental to state here and they can be found in Ref.[3]. However, they can help to visualize the plots that in the next section are shown. Therefore, below the equations that the functions recalls in Eq.13, Eq.12 and Eq.11 are stated:

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & : \tau \geq 1 \\ -\frac{1}{4}\left[\ln \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & : \tau < 1 \end{cases} , \tag{14}$$

$$A_f^H = 2\tau[1 + (1-\tau)f(\tau)] \quad , \quad A_W^H(\tau) = -[2 + 3\tau + 3\tau(2-\tau)f(\tau)] \quad , \tag{15}$$

$$A_f^H(\tau,\lambda) = 2N_c^f \frac{Q_f(I_{3f} - 2Q_f \sin^2\theta_W)}{\cos\theta_W}[I_1(\tau,\lambda) - I_2(\tau,\lambda)], \tag{16}$$

$$A_f^H(\tau,\lambda) = \cos\theta_W \left\{\left[\left(1+\frac{2}{\tau}\right)\tan^2\theta_W - \left(5+\frac{2}{\tau}\right)\right] I_1(\tau,\lambda) + 4(3-\tan^2\theta_W)I_2(\tau,\lambda)\right\} \tag{17}$$

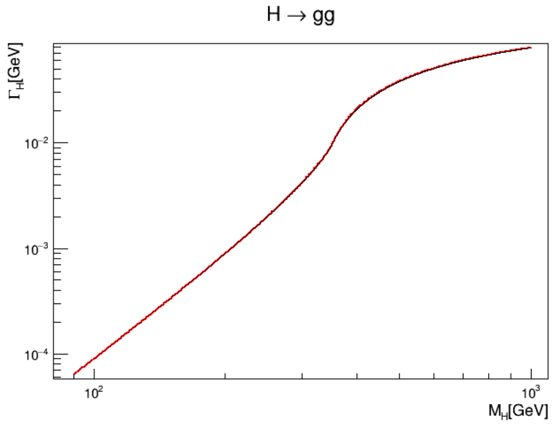
$$I_1(\tau,\lambda) = \frac{\tau\lambda}{2(\tau-\lambda)} + \frac{\tau^2\lambda^2}{2(\tau-\lambda)^2}[f(\tau) - f(\lambda)] + \frac{\tau^2\lambda}{2(\tau-\lambda)^2}[g(\tau) - g(\lambda)], \tag{18}$$

$$I_2(\tau,\lambda) = -\frac{\tau\lambda}{2(\tau-\lambda)}[f(\tau) - f(\lambda)], \tag{19}$$

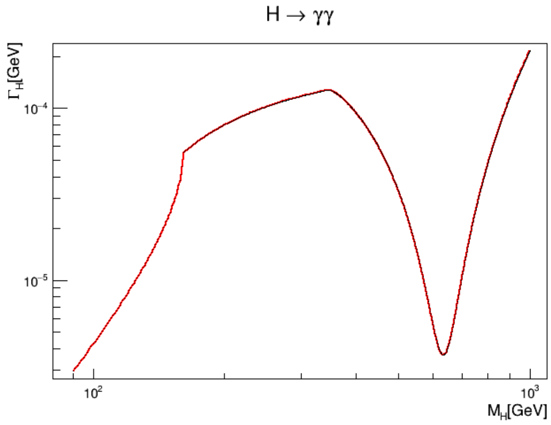
$$g(\tau) = \begin{cases} \sqrt{\tau-1}\arcsin \frac{1}{\sqrt{\tau}} & : \tau \geq 1 \\ \frac{\sqrt{1-\tau}}{2}\left[\ln \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right] & : \tau < 1 \end{cases} . \tag{20}$$

2.3.2 Plots of the deacays

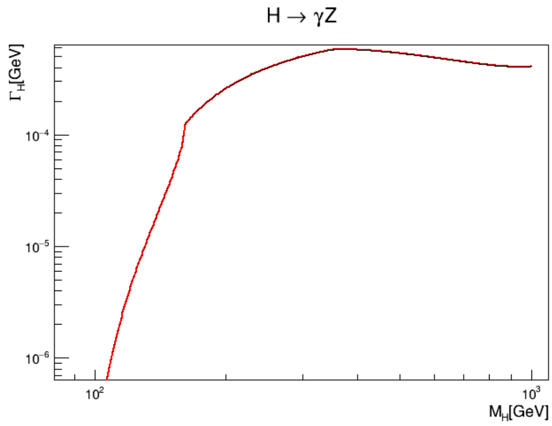
Using the 3 Function stated in the previous section (Eq.13, Eq.12 and Eq.11) it is possible to plot them as shown below:



(a) $\Gamma(H \to gg)$ vs M_H



(b) $\Gamma(H \to \gamma\gamma)$ vs M_H



(c) $\Gamma(H \to \gamma Z)$ vs M_H

Figure 7: Decay width against M_H in the case of loop-induced decay modes

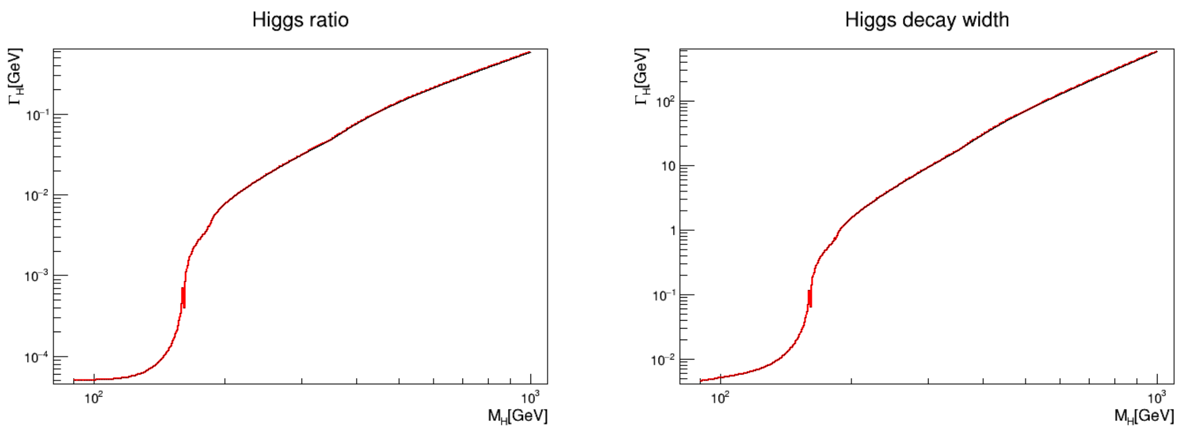
From these it is possible to understand specific behaviour of each decay. The gluon decay in Figure 7a shows that the decay width increases when M_H is greater that the double of the mass of the top quark, around ≈ 345 GeV.

Instead, for the photon decay shown in Figure 7b, one might denote that there are different point where the "trend" of the function changes rapidly, this is main due to the systems of equations written in the last section (Eq.14 and Eq.20). The first one happens approximately at the mass of the top quark around ≈ 170 GeV. The second kick happens at the double of the top quark around ≈ 345 GeV. However, after this the decay width is decreasing rapidly when the mass of Higgs boson is increasing until around ≈ 632 GeV. After this kick, it tends to infinity as the mass of the Higgs boson increases.

For the photon and Z boson decay it is possible to see it in the plot Fig.7c. There is a kick around the double of the mass of Z boson, around $\approx$ 180 GeV. After this kick it is increasing until around $\approx$ 390 GeV, where the slope starts to decrease.

3 Total decay width and branching ratio

For a greater understanding the ratio of the total Higgs decay and M_H (Γ_{total}/M_H) against M_H is plotted, as shown in Fig. 8a. Instead, in Fig. 8b, the Higgs decay width against the M_H is plotted.



(a) Γ_{total}/M_H vs M_H (b) Γ_{total} vs M_H

Figure 8: Decay width against M_H in the case of loop-induced decay modes

In the Fig. 8a it is possible to understand that the mass of the Higgs boson in which the ratio (Γ_{total}/M_H) is below 1% is around $\approx$ 220 GeV, meaning that the mass of the Higgs boson lays inside this range.

Instead, using the formula for branching ratio and plotting it against the Higgs boson mass one can found the plot in below:

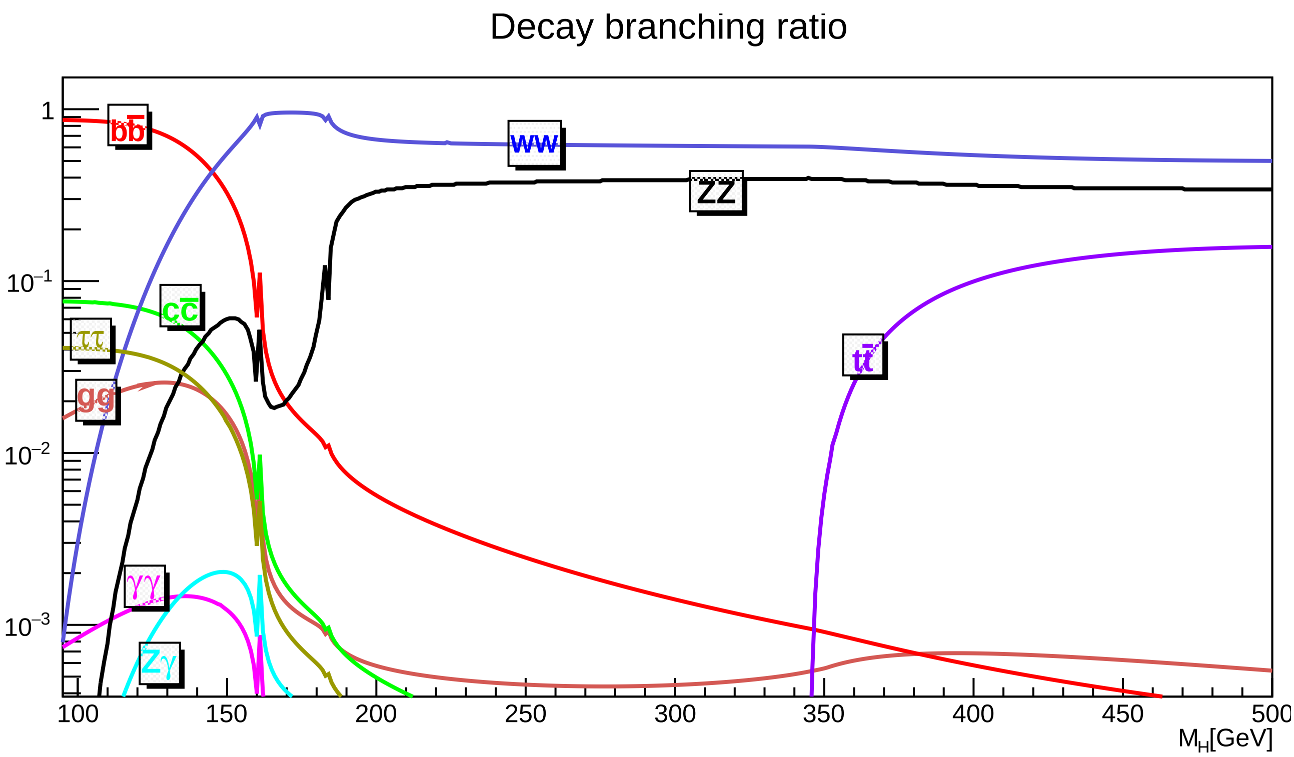


Figure 9: Branching ratio vs M_H, it is important to say that the vertical axis is dimensionless due for the Branching ratio equation

All the decays described in section 2 reside in this plot. Comparing it to the Figure 9 in Ref.[3] it is notable some differences. Especially in the decays of $c\bar{c}$, $\tau\tau$ and gg plots. This is mainly due to some correction in the decay function Γ that has been not included, especially Quantum Chromodynamics (QCD) correction for the quarks, and also Electroweak correction. Another correction that might have contributed lays in the gauge bosons decay in the off-shell, which is described here as a tree-body decay as this decay gives 4 decay body as a result.
However, the plot in Fig. 9 represents a good approximation of the BR decay, where it is possible to denote that the $H \to b\bar{b}$ decay is the most likely to occur in lower Higgs mass at around ≈ 125 GeV, which is the mass confirmed in 2012 by LHC at CERN. Calculating the branching ratio of the b quark decay it is found to be around 0.75, which confirms the theory. Instead, calculating the total decay at different M_H, as shown below:

M_H	Γ_{tot}
125	0.0073
400	27.24
600	113.77
100	520.22

it is possible to see that the total decay width increases when the M_H increases, meaning that the lifetime of the Higgs decreases at high masses. Strangely, the Higgs boson has been discovered by the $H \to \gamma\gamma$ decay which is the least probable in the plot in Fig. 9. This is mainly because this type of decay doesn't cause jets, which are distinguishable from the background.

4 Derivation of Decay width Equation

In this section is demonstrated the formula used in section 2 for the decay width of $H \to W^-W^+$ deacay. This is done by using conservation of momentum and relativistic 4-momentum notation: $p \equiv (p^\mu) \equiv (E, p_x, p_y, p_z)^T$. The momentum of the particle is represented by $H(q) \to W^-(p)W^+(k)$, leading to define the on mass shell condition as follows:

$$p^2 \equiv p \cdot p \equiv p^\mu g_{\mu\nu} p^\nu = E^2 - |\vec{p}|^2 = m^2, \tag{21}$$

where m is the particles rest mass, using the scalar product in Minkowski space. Having $q^2 = M_H^2$ and $p^2 = k^2 = M_W^2$ is possible to apply energy conservation as follows:

$$M_W^2 = p^2 = (q-k)^2 = q^2 - 2q \cdot k + k^2 = M_H^2 - 2q \cdot k + M_W^2 \tag{22}$$

and so:

$$q \cdot k = \frac{M_H^2}{2}. \tag{23}$$

Using the decay width formula, 4.18 from Ref.[3] for this particular case :

$$\Gamma = \frac{1}{2M_H} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2p^0} \frac{d^3k}{(2\pi)^3} \frac{1}{2k^0} |\mathcal{M}(p,k|^2 (2\pi)^4 \delta^{(4)}(q-(p+q)). \tag{24}$$

As the decay is only in on-shell condition using Eq. 23 the dot product of momenta is ($q \cdot q = M_H^2, p \cdot p = M_W^2, k \cdot k = M_W^2$). This means that $|M|^2$ can be factored out,

$$\Gamma = \frac{1}{2M_H} |M|^2 \Phi \tag{25}$$

where Φ is the phase space and it is:

$$\Phi = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2p^0}\frac{d^3k}{(2\pi)^3}\frac{1}{2k^0}(2\pi)^4\delta^{(4)}(q-(p+k)). \tag{26}$$

In the Higgs boson's rest frame, $q^0 = M_H$ and so $\vec{q} = 0$ which implies that $\vec{k} = -\vec{p}$ and $|\vec{k}| = |\vec{p}|$. Due to $k^2 = p^2 = m_W^2$, one has:

$$k^0 = \sqrt{M_W^2 + |\vec{k}|^2} = p^0 = \sqrt{M_W^2 + |\vec{p}|^2}. \tag{27}$$

The values of k^0 and p^0 can be substituted in Eq. 26 which gives:

$$\Phi = \frac{1}{4\cdot 4\pi^2}\int_{-\infty}^{\infty}\frac{dp^3}{\sqrt{M_W^2+|\vec{p}^2|}}\cdot\frac{dk^3}{\sqrt{M_W^2+|\vec{k}^2|}}\cdot\delta^{(0)}(q^0-(p^0+k^0))\cdot\delta^{(3)}(\vec{q}-(\vec{p}+\vec{k})). \tag{28}$$

The kronecker delta symbol has now been written as $\delta^0\delta^3$ becouse the 0th term is the energy and the other terms are components of the momentum vector. Eq. (27) is used to get rid of the δ^3 term. Now, since $\vec{q} = 0$ and $\vec{k} = -\vec{p}$

$$\delta^3(\vec{q}-(\vec{p}+\vec{k})) = \delta^3(0) = 1. \tag{29}$$

Plugging the p^0, k^0 and q^0 values into kronecker delta from the values in Eq. 27, the masses of the particles for k and p momentum are the same. Therefore is possible to rearrange in terms of p. This changes make the phase space integral equal to:

$$\Phi = \frac{1}{16\pi^2}\int_{-\infty}^{\infty}\frac{\delta(M_H - 2\sqrt{M_W^2+|\vec{p}|^2})}{M_W^2+|\vec{p}|^2}d^3p. \tag{30}$$

Changing it in spherical polar cordinates: $r = |\vec{p}|$ and $d^3p = r^2\sin(\theta)drd\theta d\phi$. Then the intengral is equal to:

$$\Phi = \frac{1}{16\pi^2}\int_{-\infty}^{\infty}\frac{\delta(M_H - 2\sqrt{M_W^2+r^2})}{M_W^2+r^2}r^2 sin(\theta)drd\theta d\phi \tag{31}$$

That can be rearranged in the following integral because there is no dependece of ϕ or θ:

$$\Phi = \frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{\delta(M_H - 2\sqrt{M_W^2+r^2})}{M_W^2+r^2}r^2dr. \tag{32}$$

The integration is achieved by using the substitution: $u = \sqrt{M_W^2 + r^2}$. Afteward taking the derivative one may achive:

$$\frac{du}{dr} = 2(M_W^2 + r^2)^{\frac{1}{2}}. \tag{33}$$

$$dr = \frac{u}{4r}du. \tag{34}$$

By applying the property of delta function ($\delta(a-x) = \delta(x-a)$) the last equation can be formulated as:

$$\delta(M_H - u) = \delta(u - M_H). \tag{35}$$

Using the delta function integration idetity ($\phi = \int_{-\infty}^{\infty} f(u) \cdot \delta(u-a)du = f(a)$) to solve the Integral. Where this identity, every value of u is 0, apart from when you have $delta(0)$, when $u = a$. Therefore using Eq. 34 and Eq. 35, one my have:

$$\phi = \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{r}{u} \cdot \delta(u - M_H)du = \frac{1}{4\pi} \frac{r_0}{M_H}, \tag{36}$$

where r_0 is the value of r when the kronecker delta function is 0. Using this to find r_0 in terms of M_H and M_W one can find:

$$0 = u - M_H, \tag{37}$$

$$M_H = 2\sqrt{M_W^2 + r_0^2}, \tag{38}$$

$$M_H^2 = 4(M_W^2 + r_0^2), \tag{39}$$

$$r_0 = \frac{M_H}{2}\sqrt{1 - \frac{4M_W^2}{M_H^2}}. \tag{40}$$

If we apply Eq. 37, Eq. 50 to Eq. 23 is possible to have:

$$\phi = \frac{1}{8\pi}\sqrt{1 - \frac{4M_W^2}{M_H^2}}. \tag{41}$$

Then using the labels $\tau = \frac{4M_W^2}{M_H^2}$ and $\beta = \sqrt{1-\tau}$, therefore the phase space can be written as:

$$\phi = \frac{\beta}{8\pi} \tag{42}$$

Therefore putting back into Eq. 25, one can obtain:

$$\Gamma = \frac{1}{2M_H}|\mathcal{M}|^2 \cdot \frac{\beta}{8\pi} \tag{43}$$

The aim now is finding the Feynman Amplitude $|\mathcal{M}|^2$. By contracting the vertex vector with the complex-conjugated polarisation vectors for W^+ and W^-, See Eq. A.9 in Ref. [5]. Therefore it can be written as:

$$\mathcal{M} = \left(2i\frac{M_W^2}{v}g^{\mu\nu}\right)(\epsilon_\mu^{\lambda_1}(p))^*(\epsilon_\nu^{\lambda_2}(k))^* \tag{44}$$

Note that λ_1 and λ_2 are the the W^+, W^- polarisation states (0, + or - respectively).This identities will help us with simplifying our derivation:

$$M_W = g\frac{v}{2}, \tag{45}$$

$$g = \frac{e}{\sin\theta_W}, \tag{46}$$

$$\alpha = \frac{e^2}{4\pi}. \tag{47}$$

Another powerfull identity is:

$$\sum_{=0,\pm}(\epsilon_\mu^\lambda(p)) * \epsilon_\nu^\lambda(p) = -g_{\mu\nu} + \frac{p_{\mu\nu}}{M^2}, \tag{48}$$

where $g_{\mu\nu}$ is a metric tensor. Therefore returning to Eq.44 the modulus squared of the Feynman amplitude is going to be:

$$|\mathcal{M}|^2 = \mathcal{M}\cdot\mathcal{M}^* = \left(2i\frac{M_W^2}{v}g^{\mu\nu}\right)\left(-2i\frac{M_W^2}{v}g^{\mu\nu}\right)(\epsilon_\mu^{\lambda_1}(p))^*(\epsilon_\nu^{\lambda_2}(k))^*(\epsilon_\mu^{\lambda_1}(p))(\epsilon_\nu^{\lambda_2}(k)). \tag{49}$$

it becomes:

$$|\mathcal{M}|^2 = \frac{4M_W^4}{v^2}g^{\mu\nu}g^{\mu\nu}(\epsilon_\mu^{\lambda_1}(p))^*(\epsilon_\nu^{\lambda_2}(k))^*(\epsilon_\mu^{\lambda_1}(p))(\epsilon_\nu^{\lambda_2}(k)). \tag{50}$$

Using metric tensors with dummy variables assigning the ϵ's a new variable. One can obtain:

$$\sum_{1,\lambda_2=0,\pm}|\mathcal{M}|^2 = \sum_{1,\lambda_2=0,\pm}\frac{4M_W^4}{v^2}g^{\mu\nu}g^{\sigma\gamma}(\epsilon_\mu^{\lambda_1}(p))^*(\epsilon_\sigma^{\lambda_1}(p))(\epsilon_\mu^{\lambda_2}(k))^*(\epsilon_\gamma^{\lambda_2}(k)). \tag{51}$$

Using the identity shown in Eq.44, afterwards we substitute Eq.45 and Eq.46 into v^2 we have a new value. Applying this into our equation gives:

$$\sum_{1,\lambda_2=0,\pm}|\mathcal{M}|^2 = \frac{e^2M_W^2}{\sin^2(\theta_W)}g^{\mu\nu}g^{\sigma\gamma}[-g_{\mu\sigma} + \frac{p_\mu p_\sigma}{M_{2W}}][-g_{\mu\sigma} + \frac{k_\nu k_\gamma}{M_W^2}]. \tag{52}$$

Expanding the brackets and multiply each term by the two tensors one can have:

$$\sum_{1,\lambda_2=0,\pm}|\mathcal{M}|^2 = \frac{e^2M_W^2}{\sin^2(\theta_W)}[g_{\mu\sigma}g_{\nu\gamma}g^{\mu\nu}g^{\sigma\gamma} - g_{\mu\sigma}g^{\mu\nu}g^{\sigma\gamma}\frac{k_\nu k_\gamma}{M_W^2} - g_{\nu\gamma}g^{\mu\nu}g^{\sigma\gamma}\frac{p_\mu p_\sigma}{M_{2W}} + g^{\mu\nu}g^{\sigma\gamma}\frac{(p_\mu p_\sigma)(k_\nu k_\gamma)}{M_W^4}]. \tag{53}$$

Therefore, applying contraction for the tensors one can simplify to:

$$\sum_{1,\lambda_2=0,\pm}|\mathcal{M}|^2 = \frac{e^2M_W^2}{\sin^2(\theta_W)}[g_{\mu\nu}g^{\mu\nu} - g^{\nu\gamma}\frac{k_\nu k_\gamma}{M_W^2} - g^{\mu\sigma}\frac{p_\mu p_\sigma}{M_{2W}} + g^{\mu\nu}g^{\sigma\gamma}\frac{(p_\mu p_\sigma)(k_\nu k_\gamma)}{M_W^4}]. \tag{54}$$

In the following procedure can be handy this three identities:

$$g_{\nu\mu}g^{\nu\mu} = 4, \tag{55}$$

$$g^{\mu\nu}x_\nu = x^\mu, \tag{56}$$

$$x^\mu \cdot y_\mu = x \cdot y. \tag{57}$$

By using the last 3 identities the Eq.54 becomes:

$$\sum_{1,\lambda_2=0,\pm} |\mathcal{M}|^2 = \frac{e^2 M_W^2}{\sin^2(\theta_W)}[4 - \frac{k \cdot k}{M_W^2} - \frac{p \cdot p}{M_W^2} + \frac{(p \cdot k)^2}{M_W^4}]. \tag{58}$$

However the value of $(p \cdot k)$ is still undetermined. One can find this by applying conservation laws of momentum (i.e. Ref.[4]) and so write it in terms of M_H and M_W:

$$q^2 = (p+k)^2 = p^2 + 2(p \cdot k) + k^2, \tag{59}$$

$$M_H^2 = 2M_W^2 + 2(p \cdot k), \tag{60}$$

$$(p \cdot k)^2 = \frac{M_H^4}{4} - M_H^2 M_W^2 + M_W^4. \tag{61}$$

Applying Eq.59 to Eq.61 into Eq.58 one can obtain:

$$\sum_{1,\lambda_2=0,\pm} |\mathcal{M}|^2 = \frac{e^2 M_W^2}{\sin^2(\theta_W)}[4 - \frac{M_W^2}{M_W^2} - \frac{M_W^2}{M_W^2} + 1 + \frac{M_H^4}{4M_W^4} - \frac{M_H^2}{M_W^2}, \tag{62}$$

which simplifies to

$$\sum_{1,\lambda_2=0,\pm} |\mathcal{M}|^2 = \frac{e^2 M_W^2}{\sin^2(\theta_W)}[3 + \frac{M_H^4}{4M_W^4} - \frac{M_H^2}{M_W^2}]. \tag{63}$$

Having all the variable defined in terms of mass is possible substitute the last equation into Eq.43 where it simplify it more. Following the derivation and using the following equations:

$$\phi = \frac{\beta}{8\pi} \tag{64}$$

$$\tau = \frac{4M_W^2}{M_H^2} \tag{65}$$

$$\alpha = \frac{\sqrt{2}}{\pi} G_F M_W^2 \sin^2(\theta_W). \tag{66}$$

Therefore the partial width decay becomes:

$$\Gamma = \frac{e^2 M_H^3}{(8\pi) M_W^2 \sin^2(\theta_W)}(1 - \tau + \frac{3}{4}\tau^2)\beta, \tag{67}$$

$$\Gamma = \frac{\alpha M_H^3}{16 M_W^2 \sin^2(\theta_W)}(1 - \tau + \frac{3}{4}\tau^2)\beta, \tag{68}$$

$$\Gamma = \frac{\sqrt{2} G_f M_H^3}{16\pi}(1 - \tau + \frac{3}{4}\tau^2)\beta. \tag{69}$$

Rationalising the $\sqrt{2}$, finally is possible to arrive at the partial decay width for $H \to W^+W^-$ as:

$$\Gamma(H \to W^+W^-) = \frac{G_F M_H^3}{16\sqrt{2}\pi} 2(1 - \tau + \frac{3}{4}\tau^2)\beta \tag{70}$$

The last equation (Eq.70) is similar to the partial decay width for the on-shell $H \to ZZ$ decay stated below

$$\Gamma(H \to ZZ) = \frac{1}{2}\frac{G_F M_H^3}{16\sqrt{2}\pi} 2(1 - \tau + \frac{3}{4}\tau^2)\beta. \tag{71}$$

They differ by a factor of 1/2 due to the fact that the Z bosons are identical. Multiply the 1/2 by 2 in the equation gives:

$$\Gamma(H \to ZZ) = \frac{G_F M_H^3}{16\sqrt{2}\pi} 1(1 - \tau + \frac{3}{4}\tau^2)\beta. \tag{72}$$

We can summarise the general solution therefore as:

$$\Gamma(H \to VV) = \frac{G_F M_H^3}{16\sqrt{2}\pi} \delta_V(1 - \tau_V + \frac{3}{4}\tau_V^2)\beta_V, \tag{73}$$

where $V = W, Z$ and $\delta_V = 2, 1$ as used in section 2. This is the final result for the derivation of the bosonic partial decay width.

5 summary and conclusion

In conclusion, the importance of theoretical prediction in particle physics has been shown by giving an explanation of the Higgs mechanism and how it gives mass to the other particles in the SM. By looking to the different decay channels it has been possible to build the Branching ratio plot, giving a great overview of the phenomena without the QCD and EW correction. Discovering that the $H \to b\bar{b}$ should be the common decay, it has been shown that it is around 0.74 at the relative $M_H \approx 125$ GeV. This means that there should be a 75% of probability that the Higgs boson at mass of 125 GeV decays in $b\bar{b}$. Most of the decays produce jets, meaning it is difficult to detect such decay. However, the $\gamma\gamma$ decay despite it is less probable does not produce jets. Finally, the theoretical formula has been derived for the the Higgs boson decay through bosons, proving the stability of the Standard Model.

References

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[3] L. Reina, *TASI 2011: Lectures on Higgs-Boson Physics*, arXiv:120.5504 [hep-ph]: Source: `https://arxiv.org/pdf/1208.5504.pdf`.

[4] Douglas C. Giancoli, *Physics for Scientists & Engineers* , Pearson Education Limited, 2013.

[5] A.V. Bednyakov, B.A. Kniehl , A.F. Pikelner, O.L. Veretin, *On the b-quark running mass in QCD and the SM*, Source: `https://arxiv.org/abs/1612.00660`.

A Code for the Figure 1 using ROOT

Listing 1: ROOT

```
//program for the potential of Abelian Higgs mechanism


{
g = new TF3("g", "(1-(x*x+y*y))^2-2*z",-1,1.2,-1,1.2,0,1);
g->SetTitle("Abelian Higgs Potential");
g->GetYaxis()->SetTitle("Re(#Phi)");
g->GetXaxis()->SetTitle("Im(#Phi)");
g->GetZaxis()->SetTitle("V(#Phi)");
g->GetXaxis()->SetTitleOffset(1.5);
g->GetYaxis()->SetTitleOffset(1.5);
g->Draw();


}
```

B Code for plotting the Fermion decays using ROOT

Listing 2: ROOT

```
// plotting fermions and bosons


//using namespace std;
//#include <cmath>
//#include <TMath.h>


using namespace TMath;



double MW(double x){
  double M = 80.398/x;
  double t1 = -(1-M*M)*((47/2)*M*M-(13/2)+(1/(M*M)));
  double t2 = -3*((1-6*M*M)+4*M*M*M*M)*Log(M);
```

```
  double t3 = 3*(((1-8*M*M)+20*M*M*M*M)/(Sqrt(4*M*M-1)))*
ACos((3*M*M-1)/(2*M*M*M));
// double t1 = x^2;
  return t1+t2+t3;
}
void hope(){
  TCanvas *c1= new TCanvas("c1","My ROOT plots", 1600, 700);
  c1->Divide(3,1);




  c1->cd(1);
  TF1 *f1= new TF1("f1", "0.000000656*x*3*(1.4)^2*(1-4*((1.4)^2)/(x^2))

^(1/2)", 10, 1000);
  gPad->SetLogx(1);
  gPad->SetLogy(1);
  f1->SetLineColor(kBlue+0);

  f1->SetTitle("lalala;X;Y");
  f1->Draw();

  c1->cd(2);
  TF1 *f2= new TF1("f2", "MW(x)", 10, 1000);
  f2->SetLineColor(kRed+0);
  f2->SetTitle("lalal3 ;X;Ylalala");
  f2->Draw();

  c1->cd(3);
  TF1 *f3 = new TF2("f3","10*cos(x)*sin(y)", -3, 3,0,10 );
  f3->Draw("surf2");


}
```

C Code used for plotting the boson decays

Is important to note that i have used the same code many times for the different bosonic decays, using python as a "calculator" and ROOT for plotting.

Listing 3: ROOT

```
// the program for plotting the bosons and loop induced from a txt file



{
g = new TGraph("bgg.txt");
//l = new TGraph("bcc.txt");

//gPad->SetLogx();
gPad->SetLogy();

g->SetTitle("brancing ");
g->GetYaxis()->SetTitle("#Gamma_{H}[GeV]");
g->GetXaxis()->SetTitle("M_{H}[GeV]");

g->GetXaxis()->SetTitleOffset(1.2);

g->GetXaxis()->SetTitleSize(0.04);
g->GetYaxis()->SetTitleSize(0.04);
//g->GetXaxis()->SetRange(55,1000);

g->SetLineWidth(2);
//g->SetLineColorAlpha(kBlue,1)
//g->SetLineColor(1);
//l->SetLineColor(3);
g->Draw();
```

```
//l->SetLineColor(kBlue+0);
//g->SetLineColor(kRed+0);
}
```

Listing 4: Python

```
# -*- coding: utf-8 -*-
"""
Created on Wed Dec  4 15:07:40 2019

@author: Farbod Rassouli
"""


import numpy as np
import matplotlib.pyplot as plt

def off_Shell(M_H):
    M_Z=91.876
    g=0.1825410462
    M_W=80.398
    sw=np.sqrt(1-((float(M_W)/float(M_Z))**2))
    def F(x):
        return -1*(1-(x**2))*(((47./2.)*(x**2))-(13./2.)+(1./float(x**2)))
-(3*(1-(6*(x**2))+(4*(x**4)))*np.log(x))+(3*(float(1-(8*(x**2))+
(20*(x**4)))/float(np.sqrt((4*(x**2))-1)))
*np.arccos(float(3*(x**2)-1)/float(2*(x**3))))
    return (float(g*M_H)/float(2048*((1-(sw)**2)**2)*((np.pi)**3)))
*(7-((40./3.)*(sw)**2)+((160./9.)*(sw)**4))*F(float(M_Z)/float(M_H))



def on_Shell(M_H):
    G=1.6637*10**-5
    M_Z=91.876
    d=1
```

```
    t=float(4*(M_Z)**2)/float((M_H)**2)
    b=np.sqrt(1-t)
    return (float(G*(M_H)**3*d)/float(16*np.sqrt(2)*np.pi))*
(1-t+((3./4.)*(t)**2))*b




n = 1
min = 90
max = 1000
delta = max-min
M_H_list=[min + float(j)/float(n) for j in range(n*(delta+1))]

def ZZ(M_H):
    M_Z=91.876
    if (M_H<M_Z):
        return 0
    elif (M_H<2*M_Z):
        return off_Shell(M_H)
    else:
        return on_Shell(M_H)

list1=[]
for M_H in M_H_list:
    list1.append(ZZ(M_H))

ZZ = np.column_stack((M_H_list, list1))
np.savetxt('zz.txt', ZZ, delimiter=' ')
```

D Code used for plotting the Loop induced decay

Using the same code of ROOT for boson decays for plotting, instead the python code for calculating is there:

Listing 5: Python

```
"""
Created on Wed Dec  4 20:06:35 2019

@author: farbod
"""


import numpy as np
def AHq(M_H):


    def tau(m_f, M_H):
        tau = (float(2*m_f)/float(M_H))**2
        return tau


    def fun(tau):
        if tau>=1:
            f = (np.arcsin(1./float(np.sqrt(tau))))**2
            return f
        else:
            f = -1*(1./4.)*(np.log(float(1+np.sqrt(1-tau))
/float(1-np.sqrt(1-tau)))



-(1j*np.pi))**2
            return f


    m_f=[1.4,4.75,172.5,1.777]
    Q=[2./3.,-1./3.,2./3.,-1]
    N=[3,3,3,1]
    sum = 0
    t_W= tau(80.398, M_H)
    f_W = fun(t_W)
    A_H_W = -(2+(3*t_W)+(3*t_W*(2-t_W)*f_W))
```

```
    for i in range (4):
        t = tau(m_f[i], M_H)
        f = fun(t)
        A_H_q = (2*t*(1+((1-t)*f)))
        sum += N[i]*(Q[i])**2*A_H_q

    return np.abs(sum+A_H_W)**2

def pdecay(M_H):
    G_F = 1.16637*10**-5
    M_W = 80.398
    M_Z = 91.1876
    sin_s = 1-float((M_W/M_Z)**2)
    alpha = (float(np.sqrt(2)*G_F*(M_W)**2*sin_s)/float(np.pi))
    return ((float(G_F*((alpha)**2)*((M_H)**3))/
float(128*((np.pi)**3)*(np.sqrt(2))))*(AHq(M_H)))

n = 1
M_H_min = 90
M_H_max = 1000
delta = M_H_max-M_H_min
M_H_list=[M_H_min + float(i)/float(n) for i in range(n*(delta+1))]

list1=[]
for M_H in M_H_list:
    list1.append(pdecay(M_H))

pp = np.column_stack((M_H_list, list1))
np.savetxt('pp.txt', pp, delimiter=' ')
```

E code for Branching ratio

Again has been used ROOT as a plotter and Python code as a calculator.

Listing 6: ROOT

```
//void multigraphpalettecolor()
{
    auto mg  = new TMultiGraph();
    auto gr1 = new TGraph("bbb.txt");
gr1->SetLineColor(2);
gr1->SetLineWidth(4);
    auto gr2 = new TGraph("bcc.txt");
gr2->SetLineColor(3);
gr2->SetLineWidth(4);
    auto gr3 = new TGraph("bgg.txt");
gr3->SetLineColor(4);
gr3->SetLineWidth(4);
    auto gr4 = new TGraph("bpp.txt");
gr4->SetLineColor(6);
gr4->SetLineWidth(4);
    auto gr5 = new TGraph("bpz.txt");
gr5->SetLineColor(7);
gr5->SetLineWidth(4);
    auto gr6 = new TGraph("btt.txt");
gr6->SetLineColor(8);
gr6->SetLineWidth(4);
    auto gr7 = new TGraph("bww.txt");
gr7->SetLineColor(9);
gr7->SetLineWidth(4);
    auto gr8 = new TGraph("bzz.txt");
gr8->SetLineColor(1);
gr8->SetLineWidth(4);
    auto gr9 = new TGraph("btau.txt");
gr9->SetLineColor(41);
```

```
gr9->SetLineWidth(4);



            mg->Add(gr4);
    mg->Add(gr3);
    mg->Add(gr2);
    mg->Add(gr1);
    mg->Add(gr5);
    mg->Add(gr6);
    mg->Add(gr7);
    mg->Add(gr8);
    mg->Add(gr9);



mg->SetTitle("Decay branching ratio; M_{H}[GeV]; #Gamma_{H}[GeV]");


mg->Draw("A");


gPad->SetLogy();
//mg->GetXaxis()->SetLimits(80.,200.);


}
```

Listing 7: Python

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
Created on Thu Dec  5 10:06:15 2019

@author: farbod
"""


import numpy as np
```

```
import fermions
import hzz
import hww
import hpp
import hpz
import ggdecay

n = 1
M_H_min = 90
M_H_max = 1000
delta = M_H_max-M_H_min
M_H_list=[M_H_min + float(j)/float(n) for j in range(n*(delta+1))]

primelist = []
list0 = []
list1 = []
list2 = []
list3 = []
list4 = []
list5 = []
list6 = []
list7 = []
list8 = []
list9 = []

def final(M_H):
    return float(fermions.fermi(3,1.4,M_H) + fermions.fermi(3,4.75,M_H) +
 fermions.top(M_H) +fermions.fermi(1,1.777,M_H) + hzz.ZZ(M_H) +
 hww.WW(M_H)
+ hpp.photondecay(M_H) + hpz.decay(M_H)
+ ggdecay.gluondecay(M_H))/float(1)
```

```
def finalRatio(M_H):
    finalRatio = float(fermions.fermi(3,1.4,M_H) +
 fermions.fermi(3,4.75,M_H) + fermions.top(M_H) +
fermions.fermi(1,1.777,M_H) + hzz.ZZ(M_H) +
hww.WW(M_H) +hpp.photondecay(M_H)
+ hpz.decay(M_H) + ggdecay.gluondecay(M_H))/float(M_H)
    return finalRatio

def BrZZ(M_H):
    return float(hzz.ZZ(M_H))/float(final(M_H))

def BrWW(M_H):
    return float(hww.WW(M_H))/float(final(M_H))

def BrPP(M_H):
    return float(hpp.photondecay(M_H))/float(final(M_H))

def BrPZ(M_H):
    return float(hpz.decay(M_H))/float(final(M_H))

def BrGG(M_H):
    return float(ggdecay.gluondecay(M_H))/float(final(M_H))

def Brbb(M_H):
    return float(fermions.fermi(3,4.75,M_H))/float(final(M_H))

def Brtt(M_H):
    return float(fermions.top(M_H))/float(final(M_H))

def Brcc(M_H):
    return float(fermions.fermi(3,1.4,M_H))/float(final(M_H))
```

```
def Brtautau(M_H):
    return float(fermions.fermi(1,1.777,M_H))/float(final(M_H))

for M in range(90,1000):
    if round(finalRatio(M) ,4) < 0.010:
        print("mass at 1%:" ,M)
    else:
        pass

for M_H in M_H_list:
    primelist.append(finalRatio(M_H))

for M_H in M_H_list:
    list0.append(final(M_H))

for M_H in M_H_list:
    list1.append(BrZZ(M_H))

for M_H in M_H_list:
    list2.append(BrWW(M_H))

for M_H in M_H_list:
    list3.append(BrPP(M_H))

for M_H in M_H_list:
    list4.append(BrGG(M_H))

for M_H in M_H_list:
    list5.append(Brbb(M_H))

for M_H in M_H_list:
    list6.append(Brtt(M_H))
```

```
for M_H in M_H_list:
    list7.append(Brcc(M_H))

for M_H in M_H_list:
    list8.append(BrPZ(M_H))

for M_H in M_H_list:
    list9.append(Brtautau(M_H))

bzz = np.column_stack((M_H_list, list1))
np.savetxt('bzz.txt', bzz, delimiter=' ')

bww = np.column_stack((M_H_list, list2))
np.savetxt('bww.txt', bww, delimiter=' ')

bpp = np.column_stack((M_H_list, list3))
np.savetxt('bpp.txt', bpp, delimiter=' ')

bgg = np.column_stack((M_H_list, list4))
np.savetxt('bgg.txt', bgg, delimiter=' ')

bbb = np.column_stack((M_H_list, list5))
np.savetxt('bbb.txt', bbb, delimiter=' ')

btt = np.column_stack((M_H_list, list6))
np.savetxt('btt.txt', btt, delimiter=' ')

bcc = np.column_stack((M_H_list, list7))
np.savetxt('bcc.txt', bcc, delimiter=' ')

bpz = np.column_stack((M_H_list, list8))
np.savetxt('bpz.txt', bpz, delimiter=' ')
```

```
btau = np.column_stack((M_H_list, list9))
np.savetxt('btau.txt', btau, delimiter=' ')


h= np.column_stack((M_H_list, list2))
np.savetxt('h.txt', h, delimiter=' ')
```