PH4110-Research Review
Extra dimensions: A unified picture for Physics beyond the Standard Model

Sayyed Farbod Rassouli
Supervisor: Dr. Nikolas Kauer, Department of Physics
Royal Holloway University of London

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Abstract

When it was discovered that a finite extra dimension to the spacetime could unify the forces of General Relativity and Electromagnetism, a new era rose for model building beyond the Standard Model. This review shows an important theoretical approach in the research literature to redefine the unified picture of physics beyond the Standard Model, and therefore a possible solution to the huge hierarchy problem among fundamental energy scales. This is done by using different shared tools between String Theory and Supersymmetry, which are applied in the context of flat extra dimension. It has been shown, by using Large extra dimensions, that the fundamental scales of physics are not fixed anymore, and can be lowered at TeV range, leading to an unified picture, where the hierarchy problem is solved. However, important solutions are found thanks to Warped extra dimensions in Anti deSitter space, leading to a different solution to the hierarchy problem.

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1 Introduction

Although the Standard Model (SM) is the most accurate description for this Universe in the current physics, it still has many discrepancies such as the huge hierarchies problems and the inconsistency with General Relativity. This review explores many combinations and shared tools of theories beyond Standard Model, such as the Minimal Supersymmetric Standard Model (MSSM), String Theory and M-theory, in the framework of extra dimensions, achieving different new models and new possibilities in the area of model-building beyond the SM.

In the next section, the complete picture of the different scales and hierarchies in the Standard Model framework (the standard paradigm) is addressed. This will give a good starting point to compare these with new models with extra dimensions described in this review. Section 2 describes the formalism to build extra dimensions, including: the Kaluza-Klein theory, compactification procedure and the advantages of compactificating on orbifold instead the usual manifolds. This provides the tools to discuss the model building required to build a general theory using extra dimensions, leading to Section 3, where these tools are combined in different layers to explain a possible D-brane scenario with Type I strings and showing the relative advantages. Section 4 is an example of embedding the MSSM theory into extra dimensions; this leads to significant results and raises important general problems to build a reliable model in the context of extra dimensions. These problems will be addressed in Section 5, where, by scaling down the three important hierarchies (GUT, Planck and String scale) thanks to large extra dimension (ADD), it will be possible to have a unified picture, which is different from the standard one described in the next Section, leading to many different possible configurations in the Brane world.

Although Sections from 2 to 5 deal with flat extra dimensions, different hints will be given throughout these Sections, since warped extra dimensions could solve many problems. These hints are regrouped and analysed in the last Section, in the framework of Randal-Sundrum (RS1) scenario, using Anti deSitter (AdS) space, explaining how this warped extra dimensions are achieved and could solve the hierarchy problem.

The standard paradigm and the Hierarchy problem

The standard paradigm of theory beyond the Standard Model can explain the energy scales which go far beyond today's colliders power. This analysis also arises the problem in the hierarchy of forces and how much they are testable by the current accelerators. By explaining each energy scale, it is possible to build a complete picture of different theories and the relative places where they fit in the hierarchy of the forces.

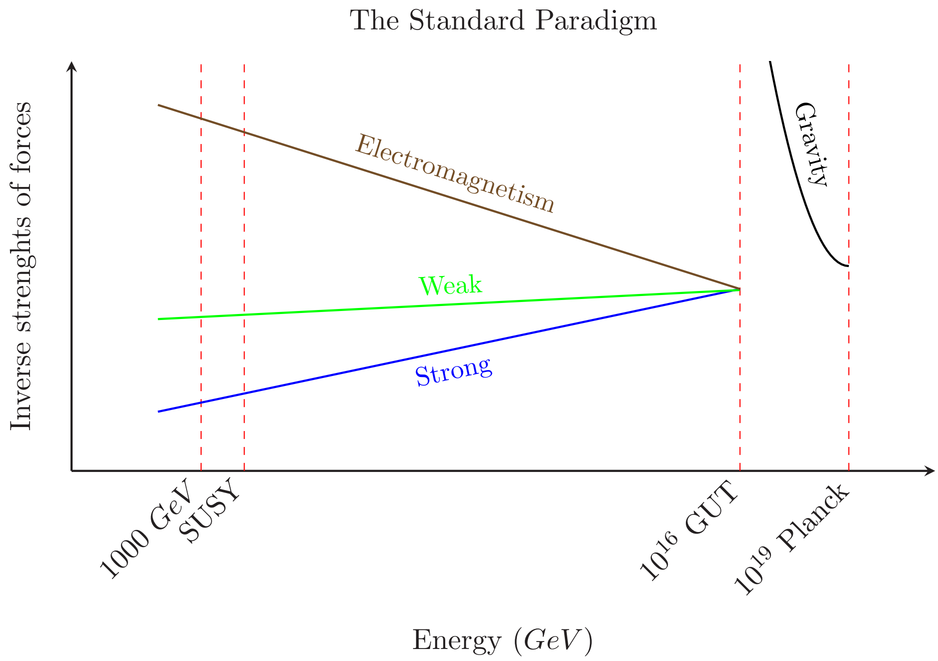


Figure 1: The different energy scales fitted in this hiearchy problem.

Starting by analysing the Grand Unified Theory (GUT) energy scale, where at high energies the gauge couplings of the Standard Model are unified together (and where, respectively, the gauge couplings corresponding to the fundamental forces are: Strong $SU(3)$, Weak $SU(2)$ and Electromagnetism $U(1)$), it is possible to extrapolate them upwards in the energy scales, both in the Standard model configuration or the respective MSSM extension. It is possible to find out that this unification of forces happens around [1]:

$$M_{\text{GUT}} \approx 2 \times 10^{16}\ \text{GeV}. \tag{1}$$

This grand unification of the fundamental gauge forces is represented in Fig. 1, in the context of other energy scales.

In contrast, Planck scale is defined by the strength of the gravitational force, where the quantum effects of gravity become strong as the gauge interactions. This scale can be determined by the

Newton's constant G_N, and therefore it is possible to define:

$$M_{\text{Plank}} = \sqrt{\frac{\hbar c}{G_N}} \approx 10^{19} \text{ GeV}. \tag{2}$$

As it is possible to denote in Fig. 1, the gravitational interaction gets stronger as the energy increments, eventually reaching the Planck scale.

In the context of physics beyond the Standard Model, the string scale is another independent relevant energy scale. This scale is ruled by the "Regge slope" α', where it is interpreted as the string tension [2]:

$$M_{\text{String}} = \frac{1}{\sqrt{\alpha'}}. \tag{3}$$

The relation between α' and the other physical parameters depends on what string model is being used. For a weakly coupled heterotic string, it is possible to set

$$\alpha' = \frac{G_N}{g^2_{\text{string}}}, \tag{4}$$

where g_{string} is the strength of string interactions. This relation must hold, since the string states that are identified as gravitons are inducing gravitational interaction at the correct strength. Unfortunately, the g_{string} is unknown, and it depends on many factors, including the String Theory model taken in consideration. Therefore, it is not possible to give a correct value for M_{string}, as for different types of scenarios this value can be quite different. This argument is based on many assumptions, but it is important to include it in this review of theories beyond the Standard Model.

It is possible to denote this huge hierarchy from the energy scales, where the energy scales of fundamental forces (GUT, Quantum Gravity and String Theory) are separated by a thirteenth order of magnitude from what we can test through the modern colliders. This is far beyond the current energy scale we can experiment today, that is the scale of electroweak symmetry breaking. By introducing Supersymmetry (SUSY), it is possible to stabilize this hierarchy from quantum correction. In order to do that, the SUSY-breaking scale (the energy scale where it is possible to detect the lightest superpartners) should be beyond the scale of electroweak symmetry breaking [3]. This is partially because SUSY is meant to protect the gauge hierarchies, otherwise it is not able to protect the quantities such as radiative correction of the Higgs mass. Another possible reason is that, in order to achieve the gauge coupling unification, SUSY-breaking scale should not be too far from the scale of electroweak symmetry breaking [4].

In conclusion, by combining these energy scales, it is possible to have a general picture of the standard paradigm for theories beyond the Standard Model, arising this huge hierarchy between these scales. This framework is drawn in Fig. 1.

2 The Kaluza-Klein Idea

A beautiful path towards a general unification of the known forces was offered by Theodor Kaluza in 1921, consisting in extending General Relativity (GR) to 4+1 dimensions [5]. This extra dimension offered a unification between GR and Electromagnetism, connecting the ten gravitational potentials g_{ik} and the four electromagnetism potentials φ_i. Moreover, the Idea of compactified dimensions was suggested by Oscar Klein in 1926 [6]. The concept of compatifing a dimension into others can be easily understood from Figure 1: the vertical dimension represents the $3+1$ dimension and the fifth dimension is *compactified* into a circle of radius R. This theory allows the existence of a 5th dimension into a $3+1$ dimension without breaking normal scale physics, and allowing the unification of GR with electromagnetism. This theory is called the Kaluza-Klein (KK) theory.

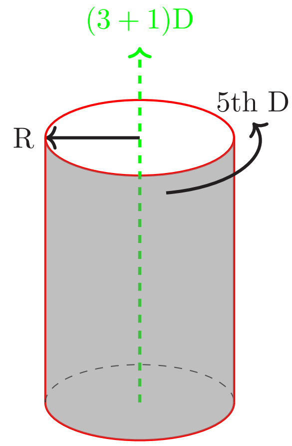


Figure 2: Ilustrative example of compactifing the 5th dimension in 3+1 dimensions.

More specifically, it is possible to understand this potential idea under the perspective of the group theory, by recognizing that the $D-$dimensional Lorentz group $SO(D-1,1)$, for any $D > 4$, is larger than the four-dimensional Lorentz group $SO(3,1)$. This implies that every single representation of the $D-$Lorentz group can be decomposed into its different representations of the four-dimensional Lorentz group. This means that every different representation of the four-dimensional Lorentz group can be identified with different particles having different spins. Therefore, different particles with different spins in four dimensions can be grouped together to be a single particle in higher dimensions. In the original case of Kaluza and Klein situation, using the Einstein gravity in five dimensions, the 5D metric tensor takes the form of:

$$G_{MN} = \left(\begin{array}{c|c} \bar{g}_{\mu\nu} & \bar{A}_\mu \\ \hline \bar{A}_\mu^T & \bar{g}_{55} \end{array}\right), \tag{5}$$

where G_{MN} is the metric with $(M,N) = 0,1,2,3,4$, having 15 independent components that are decomposed respectively in a "spin-2" symmetric tensor field $\bar{g}_{\mu\nu}$, a "spin-1" vector field $\bar{A}_\mu^T$ and a "spin-0" scalar field $\bar{g}_{55}$. The spin-two symmetric tensor is interpreted as the four dimensional graviton, whereas the spin-1 vector field is interpreted as a photon. It has been demonstrated, then, that four-dimensional gravity can be unified with electromagnetism thanks to the five-dimensional gravity. As we know, for a general unified theory we need more forces to be unified, such as: weak nuclear force, strong nuclear force, gravity and electromagnetism. Therefore, such unification will require more than five dimensions.

Following the steps of KK, this section will focus on how researchers built such ideas in the KK framework and what are the recent discoveries in this area. The following two sections will firstly describe how this compactification method is acting on the theory and will eventually take a close up about the differences between compactifying on manifold versus on orbifold.

2.1 Compactification procedure

The idea of compactifying a dimension has been introduced in the previous section and it is a vital part in the KK theory, more specifically in our KK case, where the extra dimension is a compact finite shape, instead of the normal infinite $3 + 1$ dimensions that can be perceived. It has been taken into consideration the fact that the shape is a circle in this case, therefore a circle of radius R, which is orthogonal to all dimensions. Considering M_4 to be the Minkowski space, it is possible to topologically have a space-time such as $M_4 \times S_1$, where S_1 is a circle of radius R [7]. Following this idea, it is possible to write this more generally as $M_4 \times K$, where M_4 is the usual four dimensional Minkowski spacetime and K is a δ-dimensional compact manifold. However, this type of general form is a simple factorizable structure, meaning that the extra dimension is the same in every location of the four dimensional Minkowski spacetime. This assumption can be a constraint, and other researchers have developed interesting ideas in *non*-factorizable geometries [8].

Considering the M_4 coordinate as x^μ and the K coordinate as y^i, where respectively x^μ takes value along the infinite line (4 dimensional spacetime), instead the y^i represents the specific compactification of manifold K. In the case of the aforementioned circle, a single coordinate y occurs, therefore the compactification on the circle requires an imposed periodic boundary condition of $y \to y + 2\pi R$. Once the spacetime geometry is understood, the appropriate wave functions $\Phi(x^\mu, y)$ needs to be found for this space, which must be periodic under the boundary condition. This means that $\Phi(x^\mu, y)$ must have mode expansion such as

$$\Phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi_n(x^\mu) \exp(iny/R). \tag{6}$$

This general form is consistent with the compactification symmetries written above. However, the coefficient $\phi_n(x^\mu)$ can be understood by taking into consideration that the five-dimensional field $\Phi(x^\mu, y)$ is a complex Klein-Gordon field of mass m_0, and by considering this, it is possible to find the five-dimensional action as

$$S_5 = \int d^4x \int_0^{2\pi R} dy \left[\frac{1}{2} (\partial_M \Phi)^* (\partial^M \Phi) - \frac{1}{2} m_0^2 \Phi^* \Phi \right], \tag{7}$$

where $M = (0, 1, 2, 3, 4)$, and in such way it is possible to describe with the range $0 \leq M \leq 3$ the four-dimensional Minkowski spacetime ($M = \mu$) and the compactified dimension with $x^M = y$ for $M = 4$. The analogous four-dimensional action can be derived by substituting the mode expansion in Eq. (6) into Eq. (7), and applying the integration over the compactified coordinate y.

$$\begin{aligned} S_5 =& \frac{1}{2\pi R} \int d^4x \int_0^{2\pi R} dy \Bigg\{ \frac{1}{2} \sum_{mn} \partial_\mu \phi_m^* \partial^\mu \phi_n e^{i(n-m)y/R} \\ &- \frac{1}{2} \sum_{mn} \left(\frac{-im}{R} \right) \left(\frac{in}{R} \right) \phi_m^* \phi_n e^{i(n-m)y/R} - \frac{1}{2} m_0^2 \sum_{mn} \phi_m^* \phi_n e^{i(n-m)y/R} \Bigg\}. \end{aligned} \tag{8}$$

This is the resulting equation from the substitution, where the first term arises from the kinetic term in Eq. (7) where $0 \leq M \leq 3$, and the second term comes from the kinetic term with $M = 4$. Keeping in mind that exponential wave functions having different frequencies are orthogonal, as shown below:

$$\int_0^{2\pi R} dy \, e^{i(n-m)y/R} = 2\pi R \delta_{mn}. \tag{9}$$

Then, by executing the integration of the compact extra dimension, the following four-dimensional action is given:

$$S_4 = \int d^4x \left\{ \frac{1}{2} \sum_n (\partial_\mu \phi_n^*)(\partial^\mu \phi_n) - \frac{1}{2} \sum_n \left[m_0^2 + \frac{n^2}{R^2} \right] \phi_n^* \phi_n \right\}. \tag{10}$$

Where this represents the action of infinite tower of Klein-Gordon fields ϕ_n, hence the masses are given as

$$m^2 = m_0^2 + \frac{n^2}{R^2}, \tag{11}$$

Where ϕ_n and ϕ_{-n} are degenerated for each n, therefore the lightest field is ϕ_0 with mass m_0. The index n is the discrete quantum number representing the quantized momentum in the compactified five dimension. It is possible to generalize this formula for compactified δ extra dimensions on the respective circle of radii R_i where $i = 1, ..., \delta$. Obtaining the same action as in Eq. (10) with the masses as

$$m^2 = m_0^2 + \sum_{i=1}^{\delta} \frac{n_i^2}{R_i^2}. \tag{12}$$

This example is known as *Kaluza-klein reduction* [9]. This procedure is completely general and can be applied for δ dimensions. However, the higher dimensional action S_D might contain arbitrary interaction terms between the fields. The gauge invariance and Lorentz invariance are respected in higher-dimensional action.

Starting with the mode expansion field as in Eq. (6), where the exponential are usually replaced by eigenfunctions $f(y_1, ..., y_\delta)$ depending on the manifold K of the Laplace operator, by substituting this mode expansion into the action S_D and integrating over the compactification volume, the effective four dimensional action is obtainable:

$$S_4 = \int_K d^\delta y \; S_D. \tag{13}$$

Laplace eigenfunctions keep on demonstrating orthogonality relation while performing this integral. Once having the four-dimensional Lagrangian, it is possible to read off the complete set of fields interaction and effective masses, in concordance with the effective theory in four dimensions.

From the action in Eq. (7), it is possible to understand that the Φ field has mass dimensions of $+3/2$ in five dimensions. However, it is of interest that the individual KK modes ϕ_n have mass dimension $+1$ for a four dimensional interpretation. Explaining the factor of $\sqrt{R}$ in Eq. (6), the other factor of $\sqrt{2\pi}$ arises for the kinetic term as the resulting ϕ_n fields in four dimensions will be normalized after integration over y. Therefore, in general the right prefactor will be in the form of $[\mathrm{Vol}(K)]^{-1/2}$, where K is the compactification manifold.

When in the mass formula Eq. (11) the extra dimensions are smaller compared to m_0^{-1} (that is $R^{-1} \ll m_0$), then this equation will occur:

$$m_0 \approx m_0 + \frac{n}{R}, \tag{14}$$

where the excited masses are doubly degenerated. It is important to mention that the natural units framework has been used, where R^{-1} and m_0 have the same units. The result is an infinite tower of approximately equally-spaced mass levels, having the ground state shifted by the constant m_0. The ground state, or *Kaluza-klein zero mode*, is defined by the lowest-lying state with $n = 0$. Therefore, this implicates the important argument where if during the experiments it is not possible to reach the energy level of R^{-1}, there will be no sufficient energy to excite the higher KK modes. In such a case, it will be only possible to observe the KK zero mode ϕ_0, the usual four dimensional state, where m_0 is the four dimensional mass of the four dimensional particle. Thus, to detect extra dimensions enough energy is needed in order to produce the first excited KK state, meaning that the threshold energy to detect extra dimension is $\sim R^{-1}$.

To conclude this subsection, it is important to denote two main features. First: an important signature of the extra spacetime dimensions is the appearance of an infinite tower of KK states for each known four-dimensional state. This implies that there will be a KK tower of quarks, a KK tower of photons, a KK tower of electrons and *etc.* With the ground state of each tower corresponding to the usual four-dimensional particle, it is important to note that each state in KK tower will have the same quantum numbers as those to their ground state particle. Thus, KK excitations are basically "echoes" of the extra dimensions. The second feature is the appearance of new fields along with the already known fields. This is due to the fact that it is necessary to fill the irreducible representation of the higher dimensional Lorentz group. Implying that one should see additional fields of varying spins, each of these fields will have its own KK tower states.

Thus, these conclusions hold for a compactification on smooth manifolds K and for the simple case of circle compactification. However, the second property, since it tends to require the existence of light scalar particles associated with four dimensional fields, causes phenomenological difficulties, meaning that in building such theories it has been tried to compactify in a way that is in accordance to the boundary of the experimental aspect.

2.2 Manifold versus Orbifold

Up to now, it has been discussed about the compactification procedure where K is a manifold, concentrating on the case of compactification on circles. However, to obtain realistic models, it has been found that compactification on orbifold (*i.e.*, manifold with special points like endpoints or boundaries) leads to have a chiral theory and other advantages, including at low energy phenomenologies [10–13]. In this section, the Kaluza-Klein procedure will be discussed, showing how it is modified for compactification on orbifolds, focusing on its advantages when compared to manifolds.

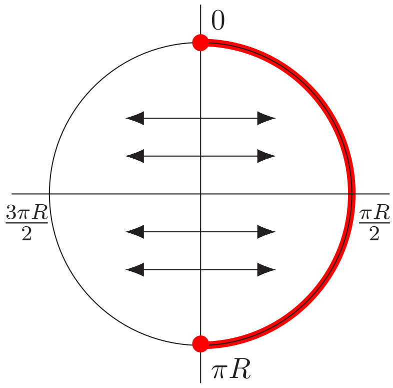


Figure 3: In red the orbifold $S^1/\mathbb{Z}_2$ with boundary points.

By considering the standard example of circle compactification [10], where K is a circle of radius R, the general math process to obtain an orbifold is to impose discrete identification between points in the manifold K, by a discrete symmetry Γ. The result quotient space K/Γ is the orbifold. The circle has the periodic boundary condition of $y \leftrightarrow y + 2\pi R$. However, imposing the additional discrete $\mathbb{Z}_2$ symmetry $\Gamma : y \leftrightarrow -y$, the resulting space K/Γ no longer consists in full fundamental range $0 \leq y < 2\pi R$ of the circle, since the points $y > \pi R$ are now identified with the points $y \leq \pi R$. This means that the fundamental range is now reduced to $0 \leq y < \pi R$ and the endpoints of this resulting segment are not identified with each other as for in the circle. These are called "fixed points" with respect to the discrete symmetry Γ. Thus, by "folding" our circle in half on itself, the resulting manifold K/Γ has the topology of a line segment of length πR. The Figure 3 shows this concept, since S^1 is used to denote the circle and $\mathbb{Z}_2$ represents any discrete $\mathbb{Z}_2$ symmetry Γ. $S^1/\mathbb{Z}_2$ is used to denote the line segment. Therefore, the presence of the special fixed-points prevents the line segment from being a manifold.

Doing this "mod out" of a manifold K by a discrete symmetry Γ and having a set of fixed points is a general procedure to create an orbifold. The number of the fixed points is called the order of the orbifold. Note that the volume of the manifold is always reduced once transformed into an orbifold.

Once the orbifold has been understood, it is possible to reconsider the previous section (the compactification procedure) and apply it on the orbifold. This means that, instead of compactifying on the usual circle, it will be necessary to compactify on the line segment $S^1/\mathbb{Z}_2$. Also, the effect of the symmetry $\Gamma : y \to -y$ has to be taken into account when following the standard procedure for compactification. This results in rewriting the KK mode expansion in Eq. (6) in terms of even eigenfunction (denoted with +) and odd eigenfunction (denoted with −) under Γ:

$$\Phi(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \left[\frac{1}{\sqrt{2}} \phi_0^{(+)}(x^\mu) + \sum_{n=1}^{\infty} \phi_n^{(+)}(x^\mu) \cos\left(\frac{ny}{R}\right) + \sum_{n=1}^{\infty} \phi_n^{(-)}(x^\mu) \cos\left(\frac{ny}{R}\right) \right]. \tag{15}$$

This is just rewriting the mode expansions. The explicit mapping between the KK modes ϕ_n in Eq. (6) and the modes $\phi_n^{(\pm)}$ in Eq. (15) are given by:

$$\phi_0^{(+)} = \phi_0, \;\; \phi_{n>0}^{(+)} = \frac{1}{\sqrt{2}}(\phi_n + \phi_{-n}), \;\; \phi_{n>0}^{(-)} = \frac{i}{\sqrt{2}}(\phi_n - \phi_{-n}). \tag{16}$$

By imposing the orbifold $\mathbb{Z}_2$ identification, therefore requiring that the five-dimensional action must be even under discrete parity symmetry $y \to -y$, if this parity along the fifth-dimension is a good symmetry, then Φ must have a definite parity. This means that, regardless of the choice, half of the KK modes will always be eliminated:

$$\text{if } \Phi \text{ is} \begin{cases} \text{even} \\ \text{odd} \end{cases}, \quad \text{then we must set} \begin{cases} \phi_n^{(-)} = 0 & \text{for all } n > 0 \\ \phi_n^{(+)} = 0 & \text{for all } n \geq 0 \end{cases}. \tag{17}$$

The choice whether Φ is odd or even is arbitrary; indeed, it mostly depends on the phenomenological needs. However, this "orbifold projection" results in the KK tower being singly degenerated, no matter the choice. Note that if Φ is chosen to be odd, then the zero-mode ground state will be lost as well.

Another important effect arises on the Lorentz structure of our higher dimensional fields. It is possible to denote this effect by considering that ϕ represents the five-dimensional gauge vector field A^M where $M = 0, 1, 2, 3, 4$. Therefore, the kinematic therm for this field takes the usual form of

$$S_5 = \int d^4x dy \left[-\frac{1}{4} F_{MN} F^{MN} \right], \tag{18}$$

where $F_{MN} = (\partial_M A_N - \partial_N A_M + \ldots)$ is the five-dimensional field strength tensor. Looking at the mixed component $F_{\mu 4} = (\partial_\mu A_4 - \partial_4 A_\mu + \ldots)$, then ∂_μ is clearly even under parity symmetry, while $\partial_4 = \partial_y$ is odd. Therefore, A_μ and A_4 must be chosen to have opposite parities. This means that, even if they are part of the Lorentz multiplet, only one of them can have zero mode. Therefore, the higher dimensional Lorentz symmetry is broken.

Once the parities for this field have been chosen, the procedure for KK reduction will follow as before. It is important to mention that the integral over the compactified dimensions must be performed on the interval of the circle and not that of the orbifold, in order to maintain the orthogonality of the respective KK modes. Even though this is a compactification on a one dimensional line segment $S^1/\mathbb{Z}_2$, these procedures and properties tend to hold disregard of the manifold and the discrete symmetry. It is important to mention that, since the five-dimensional action must be invariant under the discrete symmetry (including the possible interaction terms), then the use of an orbifold symmetry is a tool to remove certain interaction terms from a higher dimensional Lagrangian.

In conclusion, the compactification on an orbifold instead of on a manifold raises different important effects on the KK theory, including a severe reduction in the total numbers of KK modes and the elimination of the zero mode for certain fields whose transformations are not relevant for the orbifold action. This elimination leads to a procedure called "symmetry breaking by orbifolds", a very useful procedure used to break certain symmetries, in contrast to the well known Higgs mechanism (which shares some advantageous features). Thus, compactification of the orbifold plays an important role for the current research regarding the extra dimension theories.

3 D-Branes and Type I strings

In the last Section it has been described how this extra dimension can be compactified explaining the general idea of the Kaluza-klein theory, giving a good framework to better understand more recent research. In the KK framework, all the particles and forces are laying in these extra dimensions. However, it might be useful to add an extra dimension as "universal", where the extra dimension ensures that the particles follow KK excitations.

Following this idea, there are additional types of extra dimensions which may be considered. More specifically, from the developments in non-perturbative string theory, the arising of solitonic membranes, also called Dirichlet branes, can group together various gauge forces [14]. These branes can be depicted as dynamical fluctuating hypersurfaces, living in multiple D dimensions spacetime. These hypersurfaces can have multiple dimensions $p \leq D-1$. Such branes are described with notation Dp-brane where p traditionally refers to space dimensions only (this is why it is rather $p \leq D-1$ than $p \leq D$).

Taking in consideration a D3-brane, which has the power of trapping *all* the gauge forces, it is possible to better understand the idea behind these structures. In this perspective, all the gauge bosons and particles which carry gauge charges are restricted to the brane. This means that they are "trapped" in the four spacetime dimensions of the D3-brane, failing to experience any extra dimension.

They cannot propagate off the brane into the D dimensional "bulk" of the full D-dimensional spacetime. This leads to the fact that these charged particles and gauge bosons do not experience any infinite towers of KK excitations, associated with these extra dimensions in the bulk. On the flip side, the states that are neutral to these gauge forces (like the gravitons) can propagate off from the brane, experiencing the full D dimensional spacetime. Therefore, the graviton accrues KK states, whereas the particles from the Standard Model do not. This means that extra dimensions are called "gravity-only" when they are perpendicular to the branes.

An example of this manipulation about forces and particles related with branes can be found in a configuration of two D3-branes: when trapping the Gluons by setting a $SU(3)$ brane and trapping the $W^{\pm}$, Z and leptons by using another $SU(2) \times U(1)$ brane, it is possible to simultaneously trap the quarks within both branes. This means that they are trapped in the location of the respective intersection of the branes. This model of intersected branes is an important requirement in model building to give rise to the light quarks [15].

In general, the Dp-branes for $p \leq D-1$ can trap particles rising infinite towers of KK excitation belonging to the $p-3$ extra dimensions in the brane. These are called "longitudinal" extra dimensions. On the contrary, the $D-p$ dimensions in the "bulk" perpendicular to the brane will not raise any KK excitation. These are respectively called "transverse" extra dimensions [16]. However, these branes cannot be imagined as flat infinite hypersurfaces for any $p > 3$, because the $p-3$ extra dimensions in the brane must be compactified. Therefore, these branes usually have a "wrapped" shape, such as a higher dimensional cylinder, that is the usual configuration described in Section 2.1, by compactifying the extra dimensions in the brane running along the circle ahead the infinite direction of M_4. Another example is the truncate shape, formed by a semi-infinite rectangle with the infinite edges corresponding to M_4, while the finite ones correspond to the compactified space with a boundary. It is important to note that, since gravity arises from the geometry of space-time, it is impossible to "trap" gravity with any branes configuration without warping the space-time. Such theories that allow this warping are referred to as "Randall-Sundrum theories", which will be discussed in Section 6. This section represents an exception, since the whole review, except for the aforementioned section, will be about flat extra-dimensions. Therefore, it is easily notable that these structures raise many geometrical possibilities for extra dimensions model building, via extending, wrapping and/or intersecting in various ways. Then, this becomes the modern language for model building in higher dimensions.

However, it is important to realise that these D-branes have a characteristic thickness which depends on the underlying string scale. Instead, by following the work of Hořava and Witten [17] [18] (1996), the thickness of the branes is approximated to be infinitely thin, giving the advantage to incorporate the trapping of forces and particles into the higher-dimensional Lagrangian by using the Dirac δ-functions. Considering the example of the full spacetime five-dimensional action with the existence of the D3-brane locate at $y = 0$, it is possible to find:

$$S_5 = \int d^3x \int dy \{\mathcal{L}_{bulk} + \mathcal{L}_{brane}\delta(y)\}, \tag{19}$$

where $\mathcal{L}_{bulk}$ is the Lagrangian of the bulk fields in five dimensions, and, respectively, the $\mathcal{L}_{brane}$ is the Lagrangian for the trapped brane fields in four dimensions. In the case that the D3-brane traps all the gauge forces of the Standard Model, then $\mathcal{L}_{brane}$ is the Lagrangian for the Standard Model and the $\mathcal{L}_{bulk}$ is the Lagrangian for General Relativity. Thanks to the Dirac δ-function, the KK excitations for the Standard Model are eliminated. This model from Hořava and Witten forms the base for "ADD" models (by Arkani-Hamed, Dimopoulos and Dvali) [19], where the large extra dimensions are considered as a solution for the Hierarchy problem.

Finally, after showing the underlying sense of the D-branes and how it can be applied, it is important to understand the string-theoretic ideas that allow these objects to arise. The first discovery of the emergence of D-branes in String Theory is from Polchinski in 1995 [20], where he provides a setting for different fields existing in multiple extra dimensions. Before explaining the idea behind this research, it is important to have a relatively easy picture of what these strings are and how they are classified. The general idea of string theory is defined by closed strings vibrating in different modes, where each vibration represents different particles. This picture of closed strings is called Type II and heterotic strings. However, there are also string theories based on open strings and closed strings. These are called Type I strings. It is not possible to have a theory with only open strings, since such theory will always contain a string configuration where the endpoints are joined, forming a closed string.

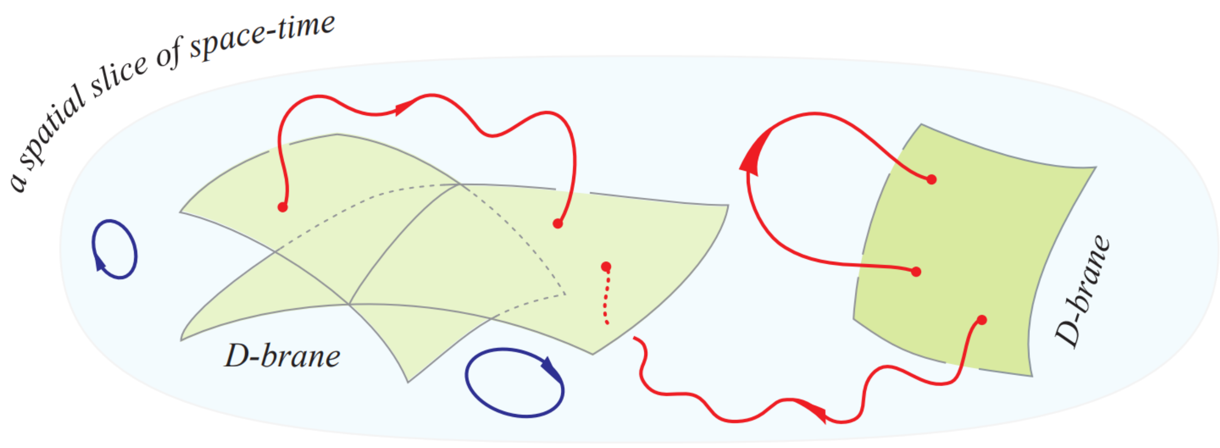


Figure 4: D-branes as boundary condition for open strings in space-time [21].

In Type I String Theory, the gauge charges associated with the particles are represented by open strings, where the actual charges are associated with the endpoints of the strings. These charges are usually called "Chan-Panton charge" [22]. It is possible to find an example in the analogy of the strong interactions, taking a meson to be an open string, where the string corresponds to the gluonic flux tube, but the endpoints of the string are associated with the quark and antiquark. By comparison, a closed string (a string without endpoints) must be neutral with respect to these charges. A possible scenario, for example, could be that all the leptons and quarks and gauge bosons are represented with open strings, due to the fact that they carry gauge charges. Anywhere else, the gravitons are neutral, therefore represented with closed strings. It is important to note that, although the Photon is neutral, it must be represented with an open string. This is due to the $U(1)$ gauge group, where the abelian nature makes the photon appear to be electrically neutral. The same happens with other particles such as the right-handed neutrino, which is neutral with respect to the gauge symmetries in the Standard Model. However, for the right-handed neutrino, it depends on which type of Type I string model it has been taken in consideration. In a model where the gauge symmetry is realised by breaking the grand unified (GUT) symmetry (like $SO(10)$ or $SU(5)$), the right-hand neutrino is going to be carried by an open string as the gauge charge which is being carried acts under the GUT symmetry. However, if the Standard Model gauge symmetry is going to be built at the string scale without passing under a GUT symmetry, it can be both open or closed string, depending on the string construction occurred.

This contrast between open and closed strings in Type I String Theory is relevant in the context of D-branes because of how they interact with them, as all the endpoints of the open strings must end on the D-brane hypersurface ("trapped" on the D-brane). However, the closed strings which have no endpoints are thus floating off the D-brane into the bulk. The sum of all these ideas in the context of extra dimensions is well represented in Fig. 4, where the intersection of different branes is represented, including both closed and open strings.

4 Embedding MSSM into Extra Dimensions

This section is fundamental, since different problems arise in embedding four dimensional theories into extra dimensions, giving the basement of the next section argument. More specifically, this example will involve the Minimal Supersymmetric Standard Model (MSSM) into five dimensions. This example will raise common problems that apply for most theories of interest [23].

The MSSM theory is based on $SU(3) \times SU(2) \times U(1) \times N = 1$ Supersymmetry (SUSY), which consists in:

- Gauge bosons: gluons, $W^{\pm}$, Z and photons,
  - They come with $N = 1$ vector supermultiplets, including the gaugino superpartners.
- Higgs fields respectively H_u and H_d,
  - They come with $N = 1$ chiral supermultiplets, including the Higgsino fields.
- Three generations of chiral fermions,
  - They come with $N = 1$ chiral supermultiplets, including both the fermions and sfermions superpartners.

However, many troubles arise when embedding this theory into five dimensions by introducing a single KK tower of identical states for each corresponding field.

The first issue that is possible to encounter concerns the structure of MSSM theory itself. This comes from the five-dimensional theory that must have at least the same supersymmetry of MSSM. However, it is possible to realise, by taking in consideration that a single spin $3/2$ gravitino in five dimensions transforms in two spin $3/2$ gravitino in four dimensions, that the $N = 1$ supersymmetry in five dimensions is transformed in $N = 2$ supersymmetry in four dimensions. This is a typical reflection of the Lorentz structure, that is expanded in the higher extra dimensions compared to the structure in four dimensional one, therefore the supersymmetry will follow this path, too.

Another problem can be found in the *chirality* nature of MSSM and is inherited from the Standard Model [24]. The chirality concept arises when the mirror image of an object cannot be superimposed onto it; these two objects are respectively called right-handed and left-handed. This object can be anything, such as a group or a system and *etc.* This property of asymmetry is an issue, since in five dimensions it is not possible to achieve chirality. This is due to the Lorentz algebra, where the Euclidean Lorentz rotation group $SU(D)$ contains chiral spinor representation only when D is even. Therefore, it is possible to achieve chirality only with even numbers of spacetime dimensions.

Analogous to the reason why the supersymmetry is expanded, a single fermion in five dimensions will be two fermions in four dimensions. The chiral fermions and its chiral conjugate have opposite chirality, thus five dimensional theory cannot be chiral.

However, the zero-mode theory (MSSM) must be $N = 1$ supersymmetric and chiral, on the other hand the field content in all the KK excitation modes must be $N = 2$ supersymmetric and chiral. Therefore, to embed the MSSM in higher dimensions, instead of repeating the $N = 1$ vector supermultiplet at each excited KK states, a new $N = 1$ chiral supermultiplet needs to be introduced for each excited KK level. This produces a $N = 2$ vector supermultiplet for all the exited singular KK levels. The similar procedure is applied for each Higgs field, producing an $N = 2$ hypermultiplet for each KK level. The story is not different for the fermions where, by introducing a chiral supermultiplet (so called chiral conjugate), it is possible to achieve a non chiral $N = 2$ hypermultiplet for each excited KK level. Therefore, at zero mode field they will maintain the original four dimensional chiral supermultiplet according to the MSSM theory.

Therefore, this results in a situation where the field content for each KK level is larger and different from the field content at zero modes. This means that, for the embedding procedure, the only way to achieve the embodiment is by compactifying on orbifolds, as discussed in Section 2.2. This is done in this specific case by compactifying on a line segment rather than on a circle. It is needed to choose the orbifold in a way that all the MSSM fields are chosen to be even while all the fields introduced beyond MSSM are chosen to be odd, with respect to the $\mathbb{Z}_2$ orbifold symmetry. By doing this, it is assured that at low energy, and therefore at the observable scale, the theory behaves purely as MSSM, due to the fact that extra fields beyond the MSSM do not have zero modes.

This case reassembles many other typical cases, and it is possible to generalize why this compactification on obifold is needed instead of a compactification on a higher dimensional manifold: the first reason is because the *chirality* is needed at the level of zero modes; It is possible to achieve this by the chiral orbifold projection that removes the chiral conjugate states. The second reason is the decrease of the extended supersymmetry arising by embedding in higher dimensions, leaving with $N = 1$ supersymmetry at zero mode level.

This results in a quasi-symmetry-breaking of the supersymmetry and a break in the non chiral spectrum into a chiral one.

5 Beyond the standard paradigm

Traditionally, extra dimensions were always thought to be quite small, around the Planck scale. This is due to naturalness, since extra dimensions emerge primarily in String Theory containing gravity, whose natural scales are at Planck scales. This means that the corresponding KK excitations will be essentially unobservable of the order $\sim 10^{19}$ GeV. For such reasons, there was not a lot of consideration for these extra dimensions theories because they would not impact the low-energy world. In particular, they could not alter the fundamental picture of theories beyond the Standard Model discussed in Section 1.

However, this started to change drastically around 1990, where high-energy theorists started to consider that these extra dimensions where not so small [3, 17–19, 25]. Therefore, the KK states were not so heavy, causing a relevant and direct effect on the physics beyond the Standard Model. It has been found that large extra dimensions (ADD) could change the fundamental high energy scales of physics, therefore energy scales such as the GUT scale and the Planck scale are not fixed anymore at high energies. Thanks to a reasonable size of extra dimensions, this high energies scales can be altered, and this is leading to a new set of possibilities for physics beyond the Standard Model [26]. Therefore, in brane the extra "longitudinal" dimensions could be large as the inverse of TeV, where the extra "transverse" ones can be large as tenths of a millimeter [27]. In the next three sections it has been described how this high energies scales could be changed, finally leading to a new paradigm different from the standard one.

5.1 Lowering the GUT scale

Starting from the GUT scale, in the framework of MSSM it is possible to derive the one-loop running of the $SU(3)\times SU(2)\times U(1)$ gauge couplings by calculating the self energy diagram represented in Fig. 5. All the MSSM states can propagate in the loop, then leading to the logarithmic renormalization group equation (RGE):

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln\left(\frac{\mu}{M_Z}\right), \tag{20}$$

where μ is the energy scale and b_i is the one-loop MSSM beta-function coefficient. This equation is in accordance with the extrapolation of the GUT energy scale shown in Section 1.

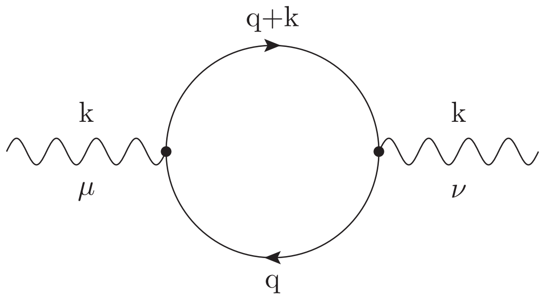


Figure 5: Self-energy diagram.

It is possible to extend this calculation into higher dimensions by considering the usual "setup" used in the previous Sections 3 and 4, where a $\delta \equiv D-4$ "Longitudinal" extra spacetime dimensions is used, each compactified in orbifolds on a circle of radius R. The R^{-1} sets the energy threshold value for the extra dimensions, which can range from TeV to the usual high energy scale. The next step is to recalculate the diagram in Fig. 5, keeping in mind the extra KK excitations are propagating in the loop for each MSSM state, arisen from the compactification into extra dimensions. Therefore, by calculating the self-energy diagram, this general result has been found:

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(\mu) - \frac{b_i - \tilde{b_i}}{2\pi} \ln\frac{\Lambda}{\mu} - \frac{\tilde{b_i}}{4\pi}\int_{r\Lambda^{-2}}^{r\mu^{-2}} \frac{dt}{t}\left\{\vartheta_3\left(\frac{it}{\pi R^2}\right)\right\}. \tag{21}$$

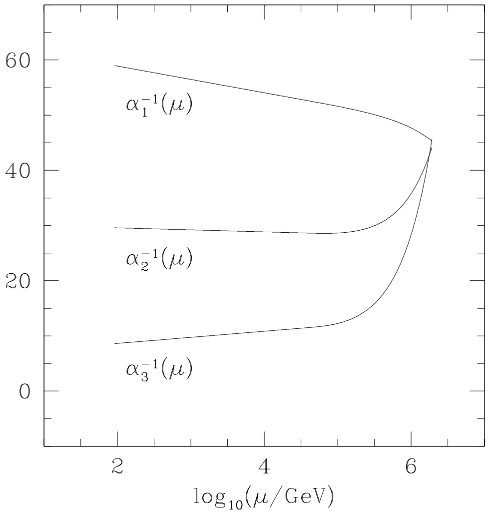
Where $\tilde{b_i}$ is the beta-function coefficient representing the matter content at the KK states. The r represents the numerical coefficient $r \equiv \pi(X_\delta)^{-\frac{2}{\delta}}$ where $X_\delta \equiv 2\pi^{\delta/2}/\delta\Gamma(\delta/2)$ is needed as an overall normalization, corresponding to the unit sphere volume of δ dimension. Finally, the Jacobi theta-function $\vartheta_3(\tau) \equiv \sum_{n=-\infty}^{\infty} \exp(\pi i \tau n^2)$ describes the sum over the KK states.

This Equation (21) can be simplified for most cases of interest, regarding energies scales above R^{-1}. By taking in consideration $\Lambda \gg R^{-1}$ and calculating the Jacobi-theta-function integral explicitly, it is possible to obtain:

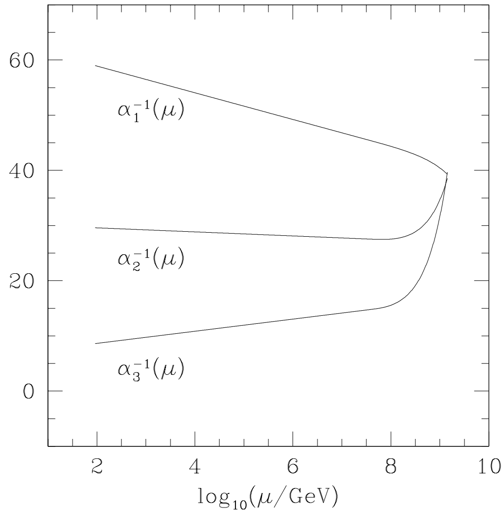
$$\alpha_i^{-1}(\Lambda) \approx \alpha_i^{-1}(R^{-1}) - \frac{b_i - \tilde{b_i}}{2\pi}\ln(\Lambda R) - \frac{\tilde{b_i} X_\delta}{2\pi\delta}\left[(\Lambda R)^\delta - 1\right]. \tag{22}$$

This simplified RGE equation is not only valid for the limit $\Lambda \gg R^{-1}$, but has been proven to hold for $\Lambda R \approx 1$ as well. The power law evolution that differs against the logarithmic evolution, related to the gauge couplings, is the difference between the Eq. (22) and Eq. (20). Therefore, the presence of extra dimensions changes the evolution of the gauge couplings [1].

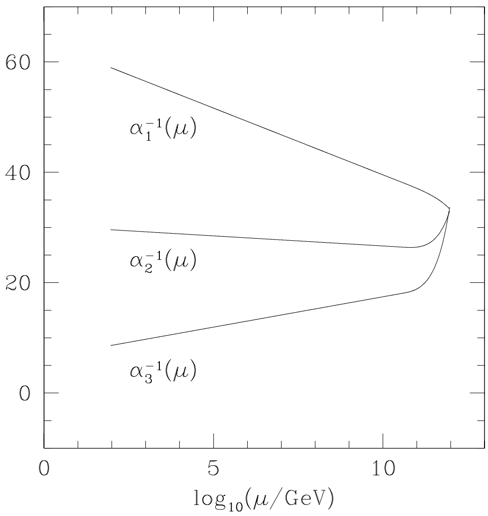
However, despite the drastic change of the gauge couplings by introducing new extra dimensions, the important outcome is that the *unification* of the gauge couplings are preserved like in Fig. (1). This means that it is possible to lower the GUT scale by changing the radius R of the extra dimensions.



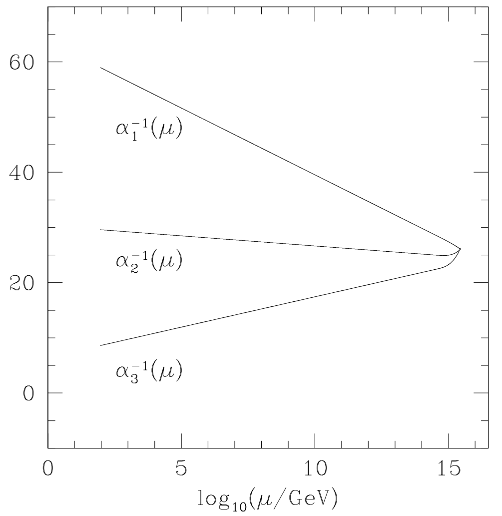
(a) $R^{-1} \approx 10^5$ GeV



(b) $R^{-1} \approx 10^8$ GeV



(c) $R^{-1} \approx 10^{11}$ GeV



(d) $R^{-1} \approx 10^{14}$ GeV

Figure 6: Unification of gauge couplings with a single extra dimensions of different radius R^{-1} [23].

In Fig. (6) it is possible to see different lower GUT scales by setting a single extra dimension and different radius R^{-1}. The unification of gauge couplings holds for multiple extra dimensions as well, since with the increasing of δ there is very little effect of increasing rate where the unification happens. It is important to note that not all embedding of MSSM into higher dimensions will result in acceptable gauge coupling unification. This mostly depends on the number of fermions which propagate in the extra dimensions and the number of Higgs fields experiencing the extra dimensions.

In conclusion, this type of unification scenario makes it possible to change the energy scale of

GUT, bringing it to intermediate energy scales or even as low as the TeV energy. This brings new possibilities and application for theories beyond the Standard Model changing the four dimensional GUT at 10^{16} GeV to higher dimensional GUT theory closer to low energy scales [23].

5.2 Lowering the Planck scale

The Planck scale as explained in the Section 1 is defined by Newton's constant G_N as shown in Eq. (2). However, when having $D-4$ extra dimensions, it is possible to associate the four dimensional Newton's constant $G_N^{(4)}$ with the D-dimensional Newton's constant:

$$G_N^{(D)} = R^{D-4} G_N^{(4)} = R^{D-4} M_{\text{Planck}}^{-2}. \tag{23}$$

It is important to note that the mass dimension of $[G_N^{(D)}]$ is equal to $(\text{mass})^{2-D}$. Thus, in $D-4$ extra dimensions, the new fundamental constant of nature is $G_N^{(D)}$. By changing the value of R, it is possible to change the Planck scale M_{Planck} [27]. Therefore, it is possible to define the higher dimensional Planck scale $M_{\text{Planck}}^{(D)}$ as

$$M_{\text{Planck}}^{(D)} \equiv \left[G_N^{(D)}\right]^{1/(2-D)}, \tag{24}$$

by substituting this into Equation (23), it is possible to have:

$$\left[M_{\text{Planck}}^{(D)}\right]^{D-2} = M_{\text{Planck}}^2 / R^{D-4} \qquad \text{or} \qquad \left[M_{\text{Planck}}^{(D)}\right]^{n+2} = M_{\text{Planck}}^2 / V_n \quad . \tag{25}$$

Where $n \equiv D-4$ and the n-dimensional compactification volume are defined as $V_n \equiv R^n$. Therefore, by changing V_n it is possible to bring $M_{\text{Planck}}^{(D)}$ in a low energy scale, similar to what has been done in the previous Section. It is possible to see this by setting $R^{-1} \approx 10^{-32/n}$ TeV or the equivalent of $R^{-1} \approx 10^{32/n-19}$ meters, while using the usual $M_{\text{Planck}} \approx 10^{19}$ GeV. Finally, applying Eq. (25) for each n dimension, it results in:

$$\begin{cases} n=1: \quad R \approx 10^{13} \quad \text{meters} \\ n=2: \quad R \approx 1 \quad \text{millimeter} \\ n=6: \quad R \approx 10 \quad \text{fermi} \approx (10\ \text{MeV})^{-1}. \end{cases} \tag{26}$$

This setup is needed in order to have $M_{\text{Planck}}^{(D)}$ in the TeV range. However, the $n=1$ case must be excluded as it is heavily changing the normal scale of physics, as for example the planetary orbits. Instead, the cases with $n>2$ are more acceptable with the gravitational experiments [19]. Keeping in mind that this type of extra dimensions are transverse to the brane, as discussed in the past Sections, therefore are only of gravitational type.

Despite this important result, where the Planck scale in higher dimensions can be lowered up, this does not resolve the gauge hierarchy problem. This is due to the fact that the M_{Planck} remains at 10^{19} GeV, while obtaining $M^{(D)}_{\text{Planck}}$, where this one is obtained thanks to the large volume $V_n \equiv R^n$. Therefore, this does not solve the big number difference between the energy scales, but merely hide it, since, in order to obtain $M^{(D)}_{\text{Planck}}$ at TeV, a big compactified volume is required. This means that, from having a huge hierarchy in energy scales, after compactifying in extra dimension, the result ends up in a huge hierarchy in geometry. Nevertheless, this is an important breakthrough to solve this problem. In fact, the solution to this problem resided in warped extra dimensions; by warping the geometry, it is possible to reduce the large number without utterly deforming the compactification space [28]. This argument will be covered in the warped extra dimensions Section.

To conclude this section, it is important to give another interpretation of the Eq. (25). From a geometrically prospective, it is possible to imagine that the gravity is "leaking" from the four dimensions into the n extra dimension, perpendicularly to the brane. Changing Newton's law in $F = G^{(D)}_N M_1 M_2 / r^{2+n}$, the gravitational interaction will be weaker than expected in four dimensions. This must be compensated by the gravity being stronger in higher dimensions rather than in four ones. Loosely speaking, the higher dimensional Planck scale, where gravity acquires strength, could be lower than the four dimensional Planck scale.

5.3 Lowering the String scale

Finally, the missing piece for having a new paradigm is string theory and lowering the string scale. This is important because, in order to construct a theory where the GUT and Planck scale are reduced, it is needed to have a theory that embeds these in a higher dimensional theory, where it must be not too far from the higher dimensional reduced GUT and Planck scale. String theory is a good candidate for this task, thus, lowering the string scale is needed [25]. By recalling the perturbative heterotic string as mentioned in Section 1, it is possible to find a relation between the Planck and string scale by combining Eq. (2), (3) and (4) [29]:

$$M_{\text{string}} = g_{\text{string}} M_{\text{Planck}}. \tag{27}$$

By assumption, from grand unified theories it is possible to affirm that the g_{string} should be approximately 0.7, resulting in $M_{\text{string}} \approx 10^{18}$ GeV [2]. However, unless the perturbative heterotic string is heavily suppressed, it is not possible to lower the string scale.

This changes for strings at strong coupling, because open Type I strings at weak coupling can be described as closed heterotic strings at strong coupling [30]. Therefore, Type I strings are the right candidate to lower string scale, as many non perturbative characteristics of heterotic string theory can be studied by using weakly coupled Type I strings. Finally, the Equation (27) is not valid for the case of Type I strings, and it is possible to replace it with

$$M_{\text{string}} \sim e^{\phi/2} g_{\text{gauge}} M_{\text{Planck}}, \tag{28}$$

where ϕ is the ten dimensional dilaton field, and g_{gauge} is the Type I gauge coupling. Therefore, it is possible to lower the M_{string} by adjusting the vacuum expectation value (VEV) of the ten dimensional dilaton [31]. However, it is possible to denote that the value of the dilaton indirectly changes the values of gravitational and gauge couplings; therefore, when the value of the dilaton is changed, M_{string} and g_{gauge} respectively change, too. Nevertheless, it is possible to get around this dependence problem algebraically, therefore it is possible to directly relate M_{string} and M_{Planck} as shown in the following equation (without the dependence expressed by the dilaton):

$$M_{\text{string}} \sim \sqrt{\frac{1}{\alpha_{\text{gauge}} M_{\text{Planck}}}} V^{-1/4}, \tag{29}$$

where $\alpha_{\text{gauge}} \equiv g^2_{\text{gauge}}/(4\pi)$ and $(2\pi)^6 V$ is the normalized six dimensional compactified volume. It is important to note that all numerical factors of order one have been ignored, since only an estimation of the order of magnitude is needed.

In conclusion, in the framework of Type I strings, it is possible to reduce the string scale by having a large compactification volume V.

5.4 The Unified Picture: Brane World

Once it has been described how these scales can be affected by large extra dimensions, now it is possible to try to build a unified picture for the new paradigm, compared to the standard one shown in Section 1. This requires the right combination between extra "longitudinal" and "transverse" dimensions restricted to the respective brane, respectively used for lowering the GUT scale and the Planck scale. However, String theory requires at least six extra dimensions to be lowered, and by choosing correctly from Eq. (29) it is possible to simultaneously lower all three scales and having a grand unified scenario into Type I string theory in the absence of any high energy scales.

Therefore, to allow the embedding of a grand unification scenario into a Type I string theory, it could be possible to set the lowered GUT scale M'_{GUT} to be around 10 TeV. However, this attempt could start by identifying the α_{gauge} with the α'_{GUT} gauge couplings at the lowered GUT scale and associating the M_{String} to the M'_{GUT}. This is done by the δ extra longitudinal dimensions with common radius R. As seen in Section 5.1, these are the dimensions that cause the power-law corrections.

The next step is finding the common radius r for the last $6-\delta$ compactified dimensions. By defining the normalized compactified volume as $V \sim R^{\delta} r^{6-\delta}$ it is possible to find this relation:

$$\frac{M'_{\rm GUT}}{M_{\rm Planck}} \sim \alpha'_{\rm GUT} (M'_{\rm GUT} R)^{\delta/2} (M'_{\rm GUT} r)^{3-\delta/2}. \tag{30}$$

Considering $\delta = 1$ and $M'_{\rm GUT} = 10$ TeV requires and implies that:

$$\begin{cases} M'_{\rm GUT} \approx 20 \\ \alpha'_{\rm GUT} R \approx 1/50 \end{cases} \quad \Rightarrow \quad M'_{\rm GUT} r \approx 10^{-6}. \tag{31}$$

This implies that the radius r of the extra five dimensions should be smaller than the string length scale. This problem is resolved by changing the string description in Type I' strings, via T-duality [32, 33]. In general, a compactfied radius r of Type I theory is the "T-dual" of a corresponding compactified $r' \equiv (M^2_{\rm String} r)^{-1}$ of Type I' theory, as shown below:

$$T{-}\text{duality}: \quad M_{\rm String} r \;\; \leftrightarrow \;\; (M_{\rm String} r')^{-1}. \tag{32}$$

Therefore, using a Type I' theory with the T-duality concept, it is possible to find this correlation

$$(r')^{-1} \quad \sim \quad 10^{-6} M'_{\rm GUT} \quad \sim \quad 10 \text{ MeV}. \tag{33}$$

This is an important result because these five extra dimensions, which are transverse to the brane, are also the exact size required to lower the Planck scale. This means that the lowering action of the GUT scale requires the lowering of the Planck scale and vice versa, within the context of string theory embedding.

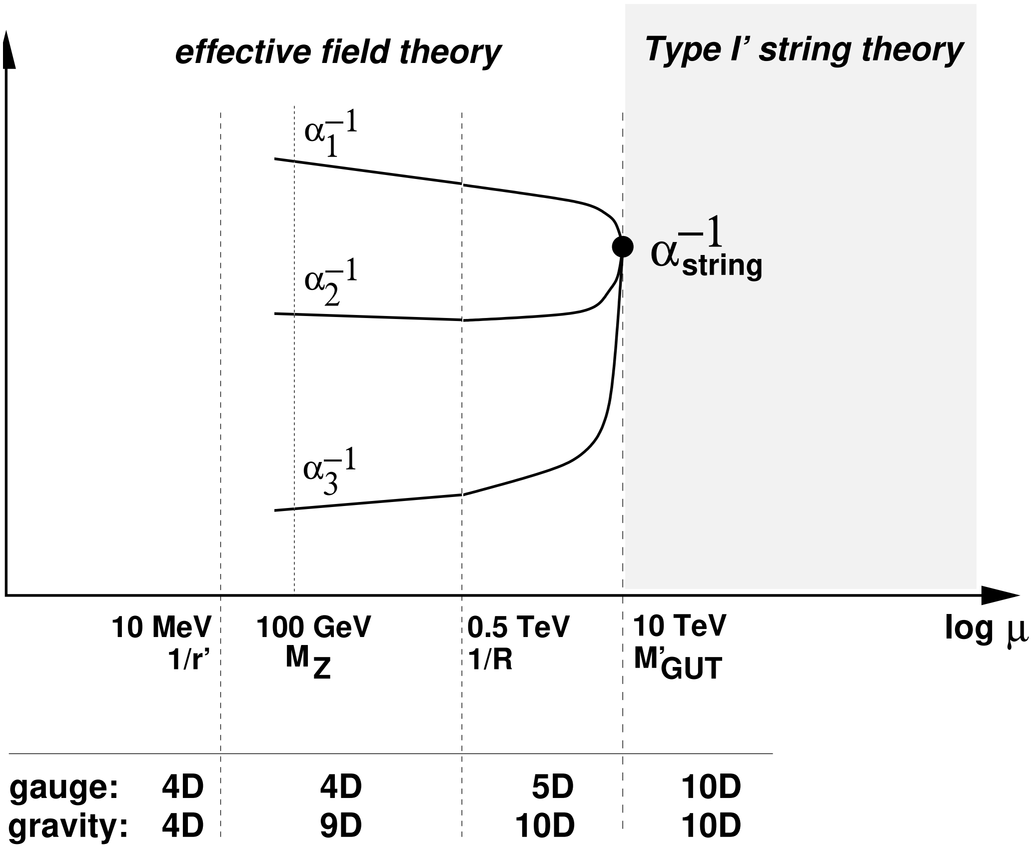


Figure 7: Sketch of the evolution of the gauge couplings within a Type I' realization of our scenario [23].

Therefore, as shown in this example, it is possible to associate the gauge couplings scale with the string scale of a Type I' at 10 TeV. This resulting scenario is sketched in Fig. 7, representing a different alternative from the standard paradigm, where the big energy hierarchy is eliminated by using large extra dimensions. In this scenario, when the physics is above the string scale at 10 TeV, then it is described with a Type I' string theory. Instead, when below the 10 TeV, it is described through a series of effective field theories, where the gravitational forces feel different numbers of spacetime dimensions. This is one of the possible configurations for a new paradigm leading to great new possibilities in this framework.

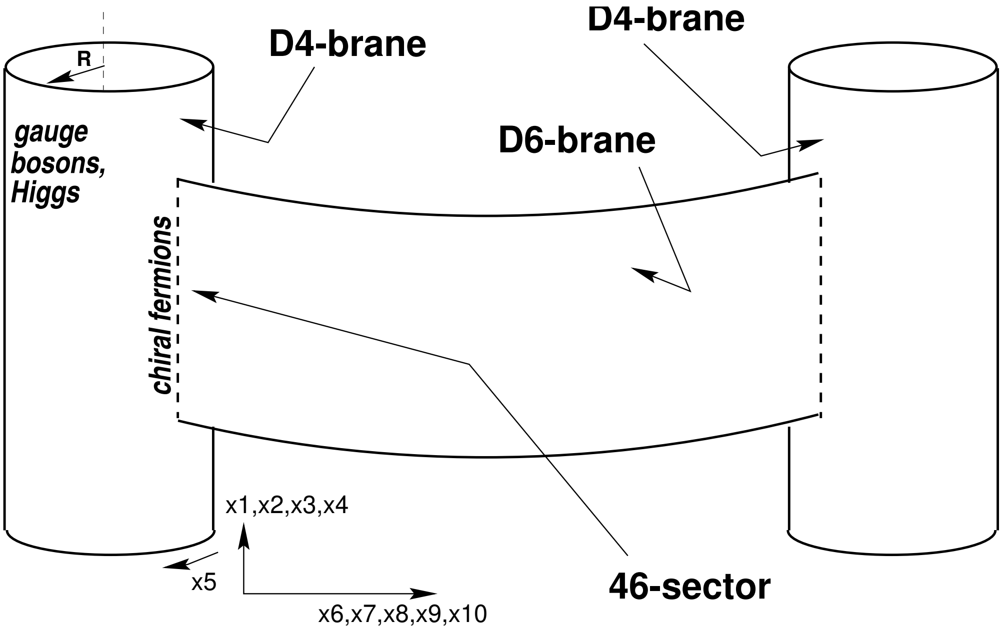


Figure 8: A D-brane configuration which can accommodate this scenario within the context of Type I' string theory [23].

The spacetime geometry associated with this model has been sketched in Fig. 8, where the observed four-dimensional world (Standard Model or MSSM) lives on the "46-sector", the brane intersection between D4-brane and D6-brane. However, at higher energies, the gauge forces feel the extra longitudinal dimension represented as a cylinder of radius R (D4-brane). It is important to note that in order to break the extended supersymmetry to $N = 1$, which introduces chirality, the circular extra dimension shown as cylinder should be replaced with a finite line segment. Also, the additional transverse direction is felt only by the gravitational interaction, lowering the Planck scale. Finally, these combinations simultaneously lead to achieve a lower string scale that ensures a robust and calculable model unifying all energy scales and forces.

This scenario is called the "brane world" and opens new possibilities for physics beyond the Standard Model [30], by new geometries and configuration of different branes, raising the potential of theoretical model building.

6 Warped extra Dimensions

Until this section, all extra dimensions have been treated as flat. However, although the picture showed in Section 5 is a good model, warped extra dimensions offer an alternative solution for the large separation of scales in the hierarchy. By considering a five dimensional model, thanks to the introduction of Anti deSitter space (AdS) with flat branes to the correspective edges, it is possible to warp this extra dimension. This leads to another possible solution of the Hierarchy problem [34].

This configuration, called the Randall-Sundrum scenario, will be addressed in the following section. Finally, the last Section shows how this scenario could resolve the large numbers problem in the hierarchy scales.

6.1 The Randall-Sundrum Scenario

By taking in consideration a 5 dimension spacetime as $x^M = (x^\mu, y)$ where $\mu = 0, 1, 2, 3$, and the spacetime indices identified by $M = (\mu, 5)$, the 5th dimension should be compactified on the orbifold S^1/Z_2, where Z_2 is the usual symmetry identified by $y = -y$. The reason to compactify on orbifold is well explained in the Section 2.2. In this model, two D3-branes are located at the two end points of the fifth dimension. And, in order to balance the branes' energy and get a flat brane metric, a negative cosmological constant on the bulk needs to be introduced, meaning that the fifth dimension would be a slice of the AdS space. Thus, keeping the branes flat will result in curving the extra dimension, which is referred to as warped extra dimension.

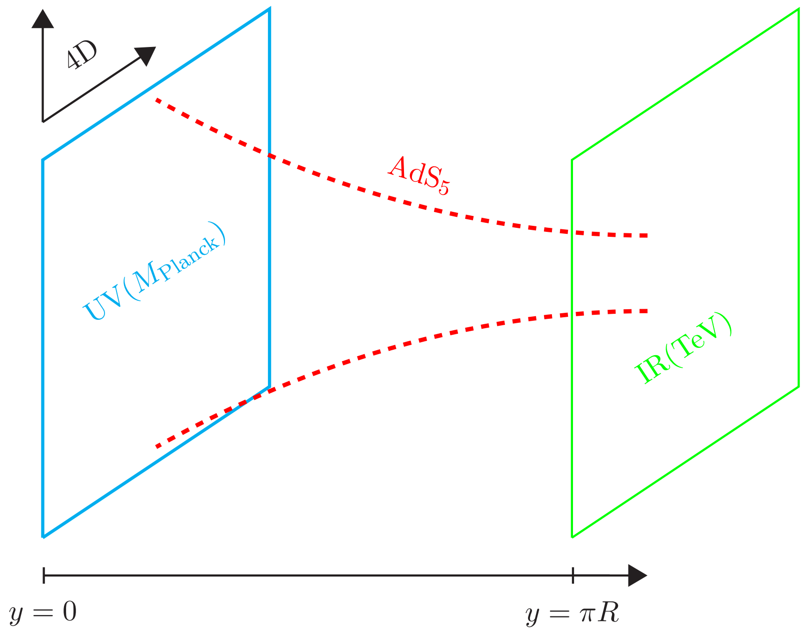


Figure 9: The Randall-Sundrum scenario.

Once this outline has been set up, it is possible to define more specifically the two $D3$-branes, which are both located at the end points of the orbifold (0 and πR); the UV brane (or Planck brane) corresponds to high energies, while IR brane (or TeV brane) corresponds to low energies. Therefore, it is possible to warp the fifth dimension via an energy per unit volume bulk cosmological constant Λ_5 in the five dimension spacetime. By adding brane tensions on both branes and tuning their values with the Λ_5 value, it is possible to obtain a zero four-dimensional cosmological constant [35]. Therefore, for such configuration, Einstein's equation takes a negative bulk cosmological constant $\Lambda_5 < 0$, obtaining an anti-deSitter space (AdS) in this warped geometry [36]. Therefore, the five dimensional metric is given by

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \equiv g_{MN}dx^M dx^N, \tag{34}$$

where $\eta_{\mu\nu} = \text{diag}(- + ++)$ represents the four dimensional metric and k is the AdS_5 curvature scale.

This slice of AdS_5 shown if Figure 9 is the Randall-Sundrum solution (RS1) [37]. This scenario is in contrast to what has been originally discussed in Section 5.2. This new model provides a low energy theory description below the standard Planck scale.

6.2 Hierarchy solution

To see how warped extra dimensions could explain the hierarchy scales, the example of the gauge hierarchy problem $m_{\text{Higgs}} \ll M_{\text{Planck}}$ has been considered, where in the context of RS1, the Standard Model particles states are confined in the IR brane. Therefore, the Higgs doublets are represented by H, a complex scalar field, having the following action:

$$S_H = -\int d^5x\sqrt{-g}\{g^{\mu\nu}\partial_\mu H^\dagger\partial_\nu H - M_5^2|H|^2 + \lambda|H|^4\}\delta(y - \pi R), \tag{35}$$

where $g \equiv \det g_{MN}$, and Higgs mass M_5 represents, in the slice of AdS_5, a value near to the five dimension cutoff scale. However, by applying the metric in Eq. (34) and performing the y integration, it is possible to obtain:

$$S_H = -\int d^4x\{e^{-2\pi kR}\eta^{\mu\nu}\partial_\mu H^\dagger\partial_\nu H - M_5^2 e^{-4\pi kR}|H|^2 + \lambda e^{-4\pi kR}|H|^4\}, \tag{36}$$

resulting in the four dimensional action for the Higgs field. By rescaling the field $H \to e^{\pi kR}H$, it is possible to achieve canonical normalization of the kinetic term in the Higgs field, as shown below:

$$S_H = -\int d^4x\{\eta^{\mu\nu}\partial_\mu H^\dagger\partial_\nu H - (M_5 e^{-\pi kR})^2|H|^2 + \lambda|H|^4\}. \tag{37}$$

Where the original mass parameter is scaled down by $e^{-\pi kR}$. This is caused by the Higgs boson residing in the IR brane ($y = \pi R$). Therefore, thanks to the new Higgs mass defined as $m_{\text{Higgs}} \propto M_5 e^{-\pi kR}$, it is possible to resolve the hierarchy problem by choosing the appropriate parameters. Therefore, by assuming $M_5 \approx k \approx M_{\text{Planck}}$ and setting $R \approx 10M_{\text{Planck}}^{-1}$, it is possible to achieve m_{Higgs} in the TeV range.

However, more in general, it is possible to see that any mass scale on the IR brane is shifted down by the amount $e^{-\pi kR}$. This leads all higher dimension operators, such as flavor changing neutral currents (FCNC), neutrino masses and proton decay, to be suppressed by the warped-down scale. A solution to this problem can be found by placing only the Higgs field on the IR brane, where the SM fermions and gauge fields can propagate in the bulk [8, 38, 39]. By using this setup, the UV brane supplies a scale high enough to suppress higher-dimension operators, while solving the fermion mass hierarchy and gauge hierarchy problem [40].

In conclusion, this is one possible setup in the context of warped extra dimensions. Unlike the method used in Section 5.2, where the huge hierarchy among scales is merely hidden in the geometry, the use of warped extra dimensions solves this problem, leading again to new different possibilities and configurations for model building in the context of physics beyond the Standard Model.

7 Conclusion

In conclusion this review has investigated how the fundamental picture of the theories beyond the Standard Model can be altered by flat and warped Extra dimensions.

By explaining the current scales of the theories beyond the standard model (GUT, Plank and String scale), it has been possible to introduce the hierarchy problem. The effect and structure of flat extra dimension could be easily understood thanks to the original Kaluza-Klein idea showing the necessary tools used in extra dimension theory, such as compactification procedure on manifolds and orbifolds. However, it has been discovered that compactifying on orbifolds leads to a severe reduction in the total number of KK modes and the elimination of the zero mode for fields that act non-trivial to the orbifold symmetry. This means that the new fields introduced by the compactifying procedure are eliminated and a symmetry breaking is induced by the orbifolds. Thanks to the embedding of the MSSM into extra dimensions, it has been proven that compactifying on orbifold is crucial in order to have a chiral theory and leaving the $N = 1$ supersymmetry at zero mode level.

Nevertheless, by intersecting multiple D-branes and using the idea of open and closed strings from Type I string theory, it was possible to accommodate different forces and particles in one theory and simultaneously satisfying each need that they require.

By regrouping all these tools and ideas in the context of large extra dimensions, it was possible to reduce the three important scales explained originally in the standard paradigm. Therefore, the lowering action of the GUT scale requires the lowering of the Planck scale and vice versa, within the context of string theory embedding, resulting in a Type I String Theory scale reduced to 10 TeV, hence reducing the fundamental scales like the Planck scales high-energies of physics at TeV scale. This framework introduces the brane world and an unified picture for physics beyond the SM.

Nonetheless, by introducing the RS1 scenario in the AdS space, it was possible to solve the huge hierarchy hidden in the geometry. This was inherited by reducing the Planck scale in the context of flat extra dimension. Therefore, warped extra dimensions give another different theoretical approach to resolve the gauge hierarchies and phenomenological challenges.

Although this unified picture and new possibilities are very exciting, this leads to many open questions, such as: understanding neutrino oscillation in this framework; new experimental approaches to string phenomenology; possible ADD effects on astrophysics and cosmology. Another question resides in the role of light stable KK states, a possible candidate for dark-matter. Clearly, these are only some examples of possible application of the brane world. It is important to note that only experiments will decide if large or warped extra dimensions exist in nature, and whether the fundamental energy scales of physics are as low as TeV range. However, another important conclusion is that theories such as SUSY and String Theory are used for the tools that they offer rather than the whole theory. This leads to new possible arrangements by mixing up these tools in the context of model building.

Finally, the fundamental high energies of physics are not fixed anymore, and therefore new configurations arise in the context of extra dimensions.

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